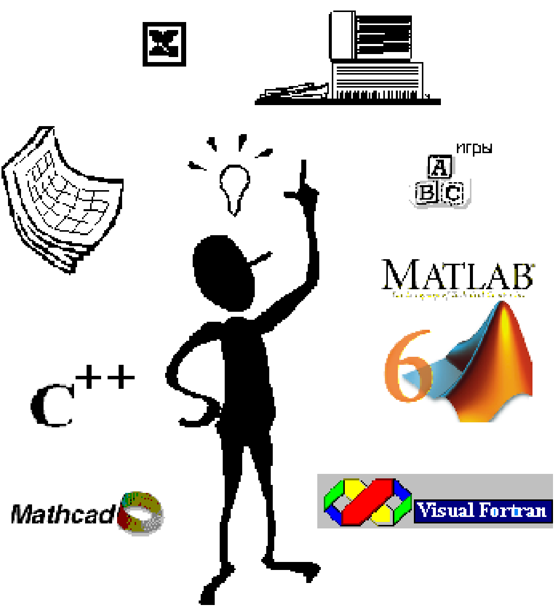
Ye.A. Gayev, B.N. Nesterenko

MATLAB
for Math and Programming

Textbook

2nd edition corrected and improved



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Reviewed by V.N. Podladchickov (Dr. Techn. Sci., Prof., National university "Kiyv Polytechnic Institute") and V.A. Kalion (PhD, Taras Shevchenko National University)

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Gayev Ye.A., Nesterenko B.N. **MATLAB for Math and Programming:** Textbook, 2nd eddition.– Kyiv: Nat. Aviation Univ, 2015. – 99 p.

This text book explains MATLAB, recently adopted by Ministry of Education for Ukrainian universities, both as valuable mathematical environment and a programming tool. Basic ideas of structured programming and theory of algorithms are illustrated by means of keynote and some original problems that allow students to quickly master developing their own programs with dialogue and graphic interface. It is intended for younger-year students as introductory modules to computer science courses.

Гаєв Є.О., Нестеренко Б.М. **MATLAB для математики та програмування:** Навч. посібник, друге видання (англійською мовою). – Київ: Нац. авіа. університет, 2015. – 99 с.

Посібник викладає MATLAB, нещодавно прийнятий Міністерством освіти в якості базового пакету для українських університетів, одночасно як математичне середовище, так і засіб ефективної розробки комп'ютерних програм. Основні засади структурного програмування і теорії алгоритмів проілюстровані на ключових та авторських задачах, що дозволяє студентам легко створювати власні програми з діалоговим та графічним інтерфейсом. Призначений для студентів молодших курсів для вступних модулів у курсах комп'ютерних дисциплін.

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Foreword

This is the second issue of our book [19] written in accordance with curriculums of disciplines "*Programming*" and "*Algorithmic Languages and Programming*" that are taught in National Aviation University for first year students taking part in English Language Educational Project. It presents a half of the whole course [20] while another half is devoted to an other algorithmic language such as *Pascal*, *Java*, *C* or C^{++}.

This textbook accounts for main needs of the first year students of many specialties. Because most of them are not familiar with programming, they need to get a fast and practical, rather that in-depth and universal, introduction to computer science, even the latter would be their future profession. At the same time, this course should be linked with and be helpful in learning other disciplines especially as difficult as mathematics and physics. While the students step by step get an ability to create simple and mediate computer programs, make them to visualize results, their new knowledge should immediately be applied in their other disciplines.

MATLAB suits best to this aim. The students learn main constructions of this language, master easily its plotting capabilities, distinguish numerical and symbolic, one-dimensional and two dimensional, numeric and text data types, master the structured programming with the flow control operators, and develop their own programs applicable to everything what they study. Module 3 of this book demonstrates the later with respect to mathematics.

It is quite unusual for Ukrainian universities yet to include MATLAB in courses of programming. However, this corresponds to world tendencies as this might be seen from textbooks [1,10-15,18]**.** Following them, Ministry of Education and Science of Ukraine accepted this software as the main mathematical tool for our universities.

The book is written in accordance with European Estimation System of the so called Bologna process. It consists of four Modules, each providing a logically closed portion of the material. **Module 1** gives ABCs of the MATLAB. Having mastered it, students may immediately apply computers in any other discipline they study. Note however that this use will be in a manner as if they used a simple calculator. **Module 2** provides basic ideas of structured programming in MATLAB. The students get knowledge in Programming Science and, simultaneously, become able to solve previous and new problems on a higher level of proficiency. A special attention is paid to technical aspects such as documenting programs, debugging them, developing "intellectual" programs of a dialogue type.

As already mentioned, **Module 3** demonstrates perspectives in learning and exploring mathematics that otherwise might look too abstract and tedious to some students. From another hand, a number of useful programming examples is contained there. Our teaching experience show that the students are usually impressed and inspired when their boring object of, say, Analytical Geometry become rotating or pulsating on computer screen… (see Fig. 4.6)

The book ends with optional **Module 4** where the students learn how to "dress" their programs into a Graphical User Interface, GUI. Again, the students are usually happy to complete programming in a modern Windows-like form. The latter may be quite difficult to them in other languages but is made very easily in the MATLAB. The new programming skills and knowledge are to be extended in their future programming courses taught in National Aviation University.

Authors wish to express their gratitude to the **Math Works Inc.** for their promotion of this book in a form of granting v. 7.1 of their wonderful software. We thank Mrs. Courtney Esposito for her constant attention to our work.

Topographical conventions of this book.

To contrast with regular text, MATLAB' commands and programs are typed in a smaller font *in italic* (except symbols like (,], **:** etc.). These, typed after the prompt symbols >>, mean a command that is issued from Command Line. Example:

>> *sqrt*(*2*+*sqrt*(*3*)).

Similar text without the prompt may correspond to MATLAB' reply, for example *Error: Missing operator, comma, or semicolon*. If a line of commands does not fit to page width, its continuation is placed on next line but aligned to the page' right border.

Navigation through MATLAB' menu is typed in bold and italic such as ***View*** ‣ ***Command Window***.

Keyboard keys are framed like [*Enter*]. New terms introduced are typed *italic*; their meaning is often self evident but students are advised to inquiry them in dictionaries or in specialized handbooks. The sign 👓 (glasses) labels optional materials, or that for advanced students.

Have any questions? Put them to the first author Ye_Gayev@voliacable.com.

"**While studying sciences,**
examples are more useful than rules"
Isaac Newton

Module 1: MATLAB, the mathematical environment

General module characteristics: The learning material provided here should introduce you very fast the main problems to be solved by the MATLAB which are often met in mathematics and physics. MATLAB will not require any programming skills but become your friendly guide into those disciplines.

Module structure

Micromodule 1.1. Basics of MATLAB
1.1.1. Getting started
1.1.2. Matrix arithmetic of the MATLAB
Micromodule 1.2. Plotting 2d functions
Micromodule 1.3. Numeric and symbolic calculations
1.3.1. Polynomials
1.3.2. Symbolic mathematics in MATLAB
Problems for Module 1

Micromodule 1.1. Basics of MATLAB

The software MATLAB is a problem oriented[1] computer system that allows to user to almost get rid of programming work[2]. There are the following reasons for learning MATLAB in our course: (*i*) it provides a top level standard of a computer software that future IT engineers are to know; (*ii*) MATLAB is an integration of several high level programming languages that comes to substitute the latter in future artificial intelligence systems; and finally (*iii*) MATLAB "bridges the gap" between applied computation and higher mathematics course [9]. It is so believed that mastering this software should be very profitable to the students of the first year.

[1] Not obviously oriented to entirely mathematical problems as this might be seen from its Help.
[2] Clearly, of programming work of a low level; as such, programming in MATLAB is explained in the next Module.

Yes, MATLAB may easily solve various problems of your university practice as may do MathCAD[3] and some other programs like Mathematica, Origine, Maple etc. We have chosen the MATLAB not only because it is widely used in our university, as well as in a number of universities and research laboratories around the world, but rather because it has became an effective programming system now.

It is quite difficult to start learning MATLAB (or any other mentioned software). We invite you to follow our informal examples that bring you to understanding the whole system. Do not hesitate in applying your MATLAB knowledge to other disciplines, especially to higher mathematics course, and you get a friendly guide in your every day learning!

1.1.1. Getting started

Look for the MATLAB' logo on your PC (see Fig. 1.1), and run the software. All you will learn in this module concerns versions 6.5, 6.* and, in most instances, v. 7 of the MATLAB. Figure 1.2 presents a typical appearance of the program on the screen. Several other possible windows may be evoked via menu ***View*** ▸ but, for our initial study, all they are advised to be removed except *Command Window* and *Command History Window*. Laboratory work 1 from [3] will give you a practice in researching the appearance of the program along with some simple commands issued from the *command line*.

We "communicate" with the MATLAB on a written language by means of composing commands, variables and other objects with Latin characters *a*, *b*, …, *y*, *z*, Arabic numbers *1*, …, *9*, *0*, few additional symbols as _, +, -, *, ^, % like in many other programming languages. Any name must begin with a Latin character but not from a figure. Capital and small letters are taken different (MATLAB is thus *case-sensitive*). So the names *a12* and *A12*, *b2C4* and *B2c4* are legal and different while the names like *2Bc4* (beginning with a figure!) are forgiven in the MATLAB. Prompt in the form >> in the command line invites you for entering commands. Examples of the latter have been given below and illustrated in the Fig. 1.2.

---

[3] Naming both software originates correspondingly from **Mat**rix **Lab**oratory (no 'Mathematics' in the abbreviation!) and **Math**ematical **C**omputer **A**ided **D**esign and brings into light the difference in their concepts.

Example 1.1. To calculate "two store expression" expression

$$\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}+\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}},$$

type it, as usual, in the following "line form":

>> *sqrt*(*2*+*sqrt*(*3*))/*sqrt*(*2*-*sqrt*(*3*))+*sqrt*(*2*-*sqrt*(*3*))/*sqrt*(*2*+*sqrt*(*3*))

Pressing *Enter* executes the calculation with the answer *ans = 4.0000*. Another way of calculation is also possible if we introduce auxiliary quantities *x* and *y* and separately calculate nominator and denominator:

>> *x=2*; *y=3*; *Term1=sqrt*(*x+sqrt*(*y*))/*sqrt*(*x-sqrt*(*y*)); ... (1.1)
Term2=1/ Term1; *Result= Term1+ Term2*

Result= 4.0000. In the last case, MATLAB assigns the value *2* to variable *x*, and value *3* to *y* and substitutes them into the consequent expression. The same result will be obtained:

>> *Result= 4.0000*

but no auxiliary variable *ans*[4] will be created.

Example 1.2. Calculate expression

$$(3^{-1}(\frac{1}{2})^{-2}-27^{-\frac{1}{3}})(3\cdot\frac{2}{5}-0.9).$$

Solution. Type in the command line

y=(3^ (-1)(1/2)^ (-2) -27^ (-1/3))*(3*2/5 - .9)*

press *Enter* and get the answer *y =0.3000*. Note that putting multiplication sign * between (round!) bracket is obligatory. MATLAB "complains" if expression is written grammatically incorrect: “*Error: Missing operator, comma, or semicolon*”

[4] Abbreviation *ans* (from *answer*) is used by default if no name is assigned to variable!

Example 1.3. Calculate expression

$$\sin(\frac{25\pi}{6}) - \cos(-\frac{\pi}{3}).$$

Solution. Type in the command line

z=sin(*25*pi/6*) - *cos*(*-pi/3*).

Note that the value of $\pi = 3{,}1415...$ has been automatically prescribed in MATLAB to the variable *pi*, as well as the value of $\sqrt{-1}$, imaginary unit, to ***i*** and to ***j***.

Students are advised to investigate and to master working in the MATLAB command window themselves using laboratory work 1 [3].

1.1.2. Matrix arithmetic of the MATLAB

Matrix arithmetic, i.e. operations with matrices and vectors, is the key stone of the MATLAB. Recall its name! The following material is thus very important.

To enter a matrix into the MATLAB' environment, all its elements are to be typed in within square brackets. Elements of rows are to be separated by *spaces* or by *commas*. In contrast, *semicolons* **;** are used to separate each new row. For example, the numeric matrix

$$A = \begin{pmatrix} 1.7 & 0.021 & 120 \\ 7.34 & 11.08 & 0.0078 \\ 31.14 & 17 & 42.1 \end{pmatrix}$$

may be typed in into the command line as

A=[*1.7 .021 120.* **;** *7.34 11.08 7.8e-3* **;** *3.114e+1 17.0 42.1*] (1.2)

(look Example 4 in the Fig. 1.2). Pressing *Enter* results in displaying the matrix on computer screen. It is clear now how to enter a vector-row or vector column, for example:

$$row1 = (1.7,\ 0.021,\ 120), \qquad col1 = \begin{pmatrix} 1.7 \\ 7.34 \\ 31.14 \end{pmatrix}.$$

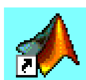


Fig. 1.1. Logo of the program to run it.

```
MATLAB
Файл  Правка  Вид  Web  Окно  Помощь
Текущий каталог: C:\MATLAB6p5\work

Команды
%-- 11/06/06  4:56 PM --%
% Here are few examples h
% through its Command W
% Example 1:
sqrt(2+sqrt(3))/sqrt(2-sqrt(3
% Example 2:
x=2; y=3; Term1=sqrt(x+sq
Term2=1/ Term1; Result= T
y=(3^ (-1)*(1/2)^ (-2) -27^ (
% Example 3:
z=sin(25*pi/6) -- cos(-pi/3)
% Example 4:
A=[1.7  .021  120. ; 7.34  11.
% Example 4a:
row1=[1.7  .021  120.], col1=

Окно команд
  Using Toolbox Path Cache.  Type "help toolbox_path_cache" for more info.

  To get started, select "MATLAB Help" from the Help menu.

>> % Here are few examples how to work with MATLAB
>> % through its Command Window and Command Line
>>
>> % Example 1:
>> sqrt(2+sqrt(3))/sqrt(2-sqrt(3))+sqrt(2-sqrt(3))/sqrt(2+sqrt(3))
ans =
    4.0000
>> % Example 2:
>> x=2; y=3; Term1=sqrt(x+sqrt(y))/sqrt(x-sqrt(y));
>> Term2=1/ Term1; Result= Term1+ Term2
Result =
    4.0000
>> y=(3^ (-1)*(1/2)^ (-2) -27^ (-1/3))*(3*2/5 - .9)
y =
    0.3000
>> % Example 3:
>> z=sin(25*pi/6) -- cos(-pi/3)
z =
    1.0000
>> % Example 4:
>> A=[1.7  .021  120. ; 7.34  11.08  7.8e-3 ;  3.114e+1  17.0  42.1]

A =

    1.7000    0.0210  120.0000
    7.3400   11.0800    0.0078
   31.1400   17.0000   42.1000

>> % Example 4a:
>> row1=[1.7  .021  120.], col1=[1.7;  7.34;  31.14];

row1 =

    1.7000    0.0210  120.0000

>>
Старт
```

Fig. 1.2. Appearance of the MATLAB with two Windows open: 1 – Command Window with the "prompt" 2; 3 – Command History Window; 4 – Menu icons.

It should be typed in by the command line correspondingly:

$$row1=[1.7\ \ .021\ \ 120.],\ \ col1=[1.7;\ \ 7.34;\ \ 31.14] \qquad (1.3)$$

A special operation has been defined for vectors (row-vectors actually) with regular numbers. Say, *x=pi : 2*pi/1000 : 3*pi* assigns to *x* 1001 elements starting from *pi* with the increment *2*pi/1000*. (Note: use semicolon ; at the end to prevent output all these numbers to the screen!)

Some special matrices may be obtained in MATLAB:

$$zeros(2,3)=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad ones(3,2)=\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$eye(4)=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The English names *zeros*, *ones* and *eye* provide hints for understanding these matrices. To get more information, ask Help from the command line, for example *help eye*.

Examples given over demonstrate that the language used by MATLAB is very close to common mathematical writing. However, what to do if a string of numbers to enter like in (1.2) or (1.3) is too long? For hyphenation, use three dots **. . .** like in the example (1.1).

Another work around lies in constructing big matrix from its parts. Separate rows (or columns) of the matrix *A* may be entered first:

row2= [*7.34 11.08 7.8e-3*]; *row3*=[*3.114e+1 17.0 42.1*]

(or *col2*=[*.021*; *11.08*; *17.0*], *col3*=[*120.*; *7.8e-3*; *42.1*]);
(putting coma or semicolon at the end depends on you wish to see results on the screen, or not). Then, the whole matrix is obtained as

A=[*row1* ; *row2* ; *row3*] or *A*=[*col1* , *col2* , *col3*]

For accessing an element of the matrix, say in second row but in third column, use its indexes in round brackets, *a*=*A*(*2,3*)= 0.0078. Such a versatile manner allows also extract any sub-matrix from *A*. Try for example:

$$A1=A(2:end\ ,\ 1:end) \qquad (1.4)$$

Such a key word *end* allows shifting elements of a vector *v* in clockwise direction in the following simple way:

>> *FirstElement*=*v*(*1*);   *v*=*v*(2 : *end*);
>> *% Getting shifted array*:
>> *v*=[*v*, *FirstElement*]
(Row-vector *v* is assumed to be already introduced into MATLAB environment, for instance *v*=[*1 2 3 4 5 6 7*]. Think how to modify commands to work with column-vectors!)

Comment: Last problem with shifting array could not be solved by other programming languages in such a simple way but as a *for-loop*. Information technology (IT) specialists are to know that *matrices*, or, more commonly in IT, numeric *arrays* are one of basic structures in any modern algorithmic language [8,9,16]. *Numbers* are particular case, an *1x1* array. However, MATLAB solves many IT problems in its own original way. Particularly, construction (1.4) solves the problem of *dynamical memory* [7] by introducing an auxiliary key word *end* denoting the size in each dimension.

Students know from linear algebra about adding, subtraction, multiplication and exponentiation of matrices. Exactly the same operations are used in MATLAB, *A+B*, *row1-row3*, *row2*col3*, *col3*row2*, *A^2*, *A^3* etc. The known restrictions to dimensions of operands are valid; otherwise one gets the warning message: "*??? Error using ==> * Inner matrix dimensions must agree*."

Although division / is not defined in the linear algebra except for numbers, *left division* / and *right division* \ have been defined in the MATLAB (see explanations for example 1.12). At the same time, the so called *operations with dot*, or *element-by-element operations*

.*   .^   ./   .

are defined in the MATLAB for operands of the same dimensions. Their sense may be explained for multiplication:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} .* \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1k} \\ b_{21} & b_{22} & \dots & b_{2k} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nk} \end{pmatrix} = \begin{pmatrix} a_{11}*b_{11} & a_{12}*b_{12} & \dots & a_{1k}*b_{1k} \\ a_{21}*b_{21} & a_{22}*b_{22} & \dots & a_{2k}*b_{2k} \\ \dots & \dots & \dots & \dots \\ a_{n1}*b_{n1} & a_{n2}*b_{n2} & \dots & a_{nk}*b_{nk} \end{pmatrix}$$

For more information request *help /*, or *help arith*, or *help slash*. Similar, any function *sin*, *cosh*, *atan*, *sqrt*, *log10*, *exp* etc. with respect to a matrix produces a new one with the function applied element-by-element (request *help elfun* for the list of all elementary functions).

There is no problem for using complex numbers in MATLAB:

```
>> (2+3j)+(3+2i)
        ans =   5.0000 + 5.0000 i

>> sqrt(j)
        ans =   0.7071 + 0.7071 i
```

(note that there is no multiplication sign between coefficient and the imaginative unit $i=j=\sqrt{-1}$). Any matrix may be composed by complex elements.

The sign *%* (look for it in the Fig. 1.2) is used for providing *comments* that are a kind of information that MATLAB does not account for but which might be helpful to program's author or user.

It is important to pay attention to formats for presentation of real numbers. For example, the numbers

2,17 0,00217 $0{,}217\ 10^1$

should be typed into the MATLAB environment as

2.17 0.00217 .217e+1

While executing laboratory work № 1 from [3], investigate setting formats *short* and *long* for numbers!

MATLAB has been "equipped" by a number of ready functions to work with arrays. Stroke behind the matrix symbol, *A*', transposes the matrix *A*, try [*1 2 3 4*] '. The function *length*(*A*) determines the lengthy dimension of *A*. An assignment [*N* , *M*]=*size*(*A*) returns number of rows to *N*, and that of columns to *M*. Operation as *A*=*diag*(*x*), with *x* a vector, forms diagonal square matrix with the elements from *x* on the main diagonal. One more service function *sum*(*A*) finds sums of elements in each column and returns a vector with them; it follows that *sum*(*sum*(*A*)) returns only one number, a sums of all matrix elements.

Micromodule 1.2. Plotting 1d functions

MATLAB has an excellent set of graphic tools. Color graphics are very engaging for students and provide wide possibilities for their work in all areas of student's practice. We begin with two easy-to-use commands before introducing the most powerful command.

To plot graphic of a function of one variable, say

$$y = \frac{\sin x}{x}, \qquad (1.5)^{5)}$$

there is no need to calculate first a table of its values as you did this in the school. Simply execute the command

>> *ezplot*('*sin*(*x*)/*x*')

and enjoy the plot of the function in the domain $x \in [-2\pi, 2\pi]$ on default. To extend the domain to, say $x \in [-5\pi, 7\pi]$, try another command format

>> *ezplot*('*sin*(*x*)/*x*', [- *5*pi*, *7*pi*]) , *axis*([- *5*pi 7*pi -.25 1.1*]) (1.6)

Resulting graphic is presented in the Fig. 1.3 but has been slightly changed by additional tools provided by the *Figure menu*: coordinate axes were drawn by pressing icon ("*InsertLine*"); the title of graph, the thickness of curve and the color of background were changed by pressing icon ("*Edit Plot*") and evoking *Figure Property Editor*. Investigate further features of the Editor and the Figure Window! Say, the icon allows printing figure on paper.

Getting help from the MATLAB lets us to summarize the format of the *ezplot*-command in the following form:

ezplot('*f*' , [*xmin*, *xmax*, *ymin*, *ymax*]).

Note also that graphics of implicit functions like hyperbola, and parametric functions like $x = 2\sin t$, $x = 7\cos t$ (ellipse) touched in the higher mathematics course may also be plotted by this command:

>> *ezplot*('*.2*x^2 - .7* y^2=1*'), or

>> *figure*, *ezplot*(' *2*sin*(*t*) ', ' *5*cos*(*t*) ')

Another easy-to-use command *fplot* may plot several functions in the same window, each graph labelled by its colour. For instance:

>> *fplot*(' [*sin*(*x*)/*x*, *x*sin*(*x*), *cos*(*x*)] ' , [-*pi* *3*pi* *-5* *6*])

---

[5] Note that this function concerns to what is called the "first remarkable limit" in the higher mathematics course.

Note that format of the command requires providing the list of functions within square brackets and inverted commas!

The function to plot not obviously may be given in an analytical form as before. For example, experiments are a constant source of table functions. In this case, one has a vector of arguments $X = [x_1.x_2,..., x_N]$ and a vector of corresponding function values $Y = [y_1.y_2,..., y_N]$. The lengths of both vectors are to be equal. As a *mathematical experiment*, we could get both vectors by calculating table of an analytically given function. For example, for the above function (1.5) let us get first the ordered vector of arguments in a domain $x \in [a,b]$

```
>> a= - pi; b=3*pi; N=1000; X=a : (b-a)/N : b;
```

and corresponding vector of function values

```
>> Y=sin(X) ./ X;
```

(note that MATLAB "complains" *Warning: Divide by zero* but presents completely correct results. This is because the singularity in $x = 0$ is removable). Now, plot the graphic by the *plot*-command:

```
>> plot( X, Y)
```

The curve you get may be less or more smooth depending on the *sampling parameter N*. The students are advised to make some experiments by varying *N* and plotting new graphs[6].

Using the vectors *X* and *Y* you already have, try also the command

```
>> comet(X ,Y) !
```

Exercise. Plot a regular polygon of *N* sides on computer screen. Solution uses ability of MATLAB to work with complex numbers. Really, let *N=5*. In this case *N*-th root of, say, *z0=1* has *N* values $z_k = e^{2\pi ki/N}$ where $i = \sqrt{-1}$ and $k = 0, 1, ..., N-1$ that are vertices of a regular polygon on complex plane. So, produce these vertices and plot their real *real* and imaginary *imag* parts:

```
>>N=5; k=1: N+1; Vertices=cos(2*pi*k/N)+i*sin(2*pi*k/N);
>> plot(real(Vertices), imag(Vertices), 'r'), axis equal
```

---

[6] Be careful however: an error may occur if you try an *N* less than previous one. To prevent it, you may introduce new variables, say *X1*, *Y1*, instead of *X*, *Y*.

Polygon will be drawn on the screen. It will be filled in by a colour specified, like in Fig. 1.4, if command *fill* is used instead of *plot*.

Further practice with plotting graphs might be found in laboratory work № 2 in [3]. About plotting 3-dimensional graphs read [1,7].

Micromodule 1.3. Numeric and symbolic calculations

Some students ask why inverted commas are used in the *ezplot*('...') but are not in the *plot* command? This is because the first command works with *symbolic argument* while the second one with numeric ones. Let us explain this in more details.

Numbers, vectors and matrices were *numeric objects*. Operations over them follow to known arithmetic algorithms. Another algorithms are required when one performs analytical transformations like the square of a sum $(a+b)^2$ into $a^2+2ab+b^2$. Result of such transformation is valid for arbitrary a and b.

It is worth to remind that algorithms of *symbolic calculations* were first developed in Kyiv, in the Institute of Cybernetics Ukrainian National Academy of Sciences and realized in electronic machine "MIR-2" in 1970th years. However, Canadian package ***Maple*** (late of the 1990th) turned to be more competitive; its algorithms were also included into the MATLAB.

Information technology comes thus to another type of data, to *text data*, that are any "word" of legal symbols [8,9,17]. It is also natural to consider vectors and matrices with text (or *symbolic*) elements. If one introduces symbolic variables

```
>> c11=' Name '; c21=' Age '; c31='City from';
```

(spaces have been inserted within apostrophes(!) (inverted commas) to make all the "words" of the same length 9), a column-vector may be created:

```
>> Student=[c11; c21; a31]
          Student =
           Name
            Age
          City from
```

It is our aim now to demonstrate many useful consequences of the new data type.

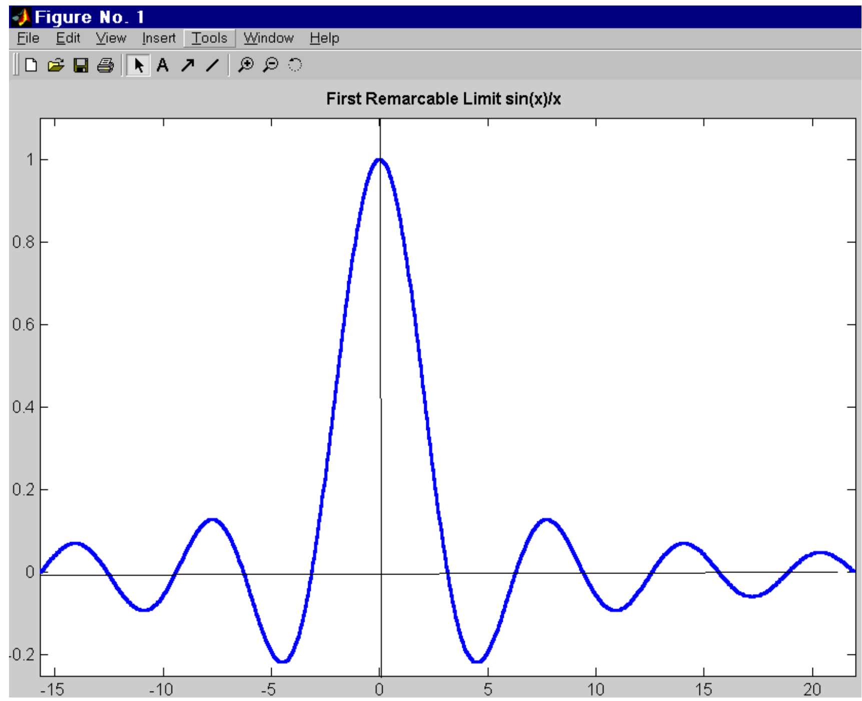


Fig. 1.3. Example of MATLAB's plotting capabilities.

1.3.1. Polynomials

Differences between numeric and symbolic objects may easily be explained by means of *polynomials* because this class of objects exists in MATLAB both as numeric and symbolic ones.

In mathematics, *polynomial* is a function of variable x, a sum of its powers

$$p(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-1} x + a_n \qquad (1.7)$$

where $\{a_k\}$ are real numbers, and n is an integer. To evaluate value of $p(x)$ at, say $x = x_0$, the latter number is to be substituted into (1.7), $p_0 = p(x_0)$.

Polynomials like (1.7) may be represented in the MATLAB environment by numeric row-vector with the coefficients,

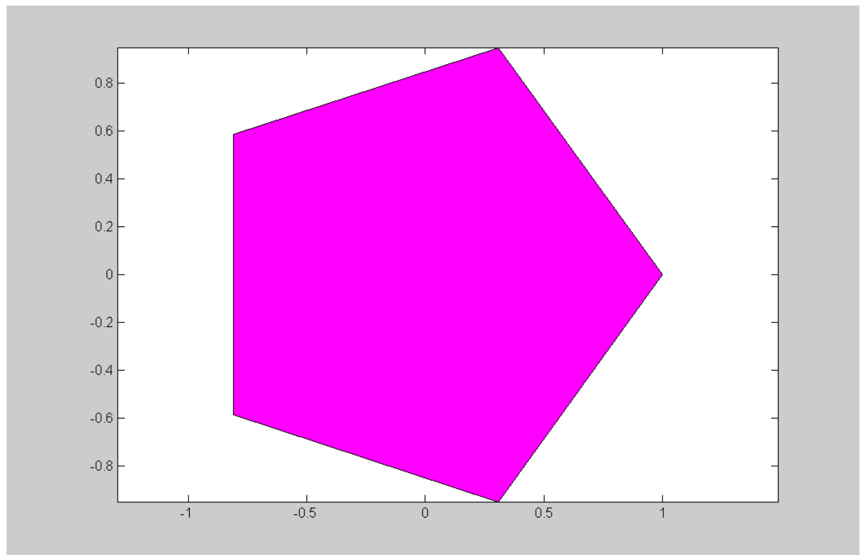


Fig. 1.4. Regular polygon of N sides on computer screen, N=5.

$$p=[a_0, a_1, a_2, \ldots, a_{n-1}, a_n]$$

(they are arranged in the order of reducing power). MATLAB's command[7] with two numeric entries *polyval*(p, x_0) calculates (1.7) for $x = x_0$. Another command *roots*(p) looks for all the roots of the polynomial. How many are them? The *main theorem of algebra* manifests that the number of roots is equal to its order n provided (i) complex roots are accounted for, and (ii) each root is accounted as many times as is the *order of the root* [2,4]. Thus, both above commands work with numeric objects.

Example 1.4. Find roots of the polynomial $p(x) = x^4 + 2x^3 + 3x^2 + 4x + 5$ and evaluate it for $x = 1$.

Solution. First, introduce the given polynomial as a numeric object into the MATLAB environment:

```
>> p=[1 2 3 4 5]
```

---

[7] Its name was derived perhaps from *polynomial value*.

Second, find all the four roots of it. Here is what will be obtained:

>> roots(p)
*ans =   0.2878 + 1.4161**i***
*0.2878 - 1.4161**i***
*-1.2878 + 0.8579**i***
*-1.2878 - 0.8579**i***

All the roots are complex numbers. Now, estimate the polynomial for $x=1$:

>> polyval(p, 1)
ans =   15

what could easily be checked from the very beginning: $p(1)=1+2+3+4+5=15$. Similar, each root may be confirmed to make polynomial almost vanish:

*>> polyval(p, 0.2878 + 1.4161**i**)*
*ans = 1.7186e-004 -1.4064e-004**i***

Exercise. Prove that (*i*) if a polynomial has a complex root of the form $x=a+bi$, the conjugate of the latter $x=a-bi$ is its root as well; (*ii*) each polynomial of an odd degree has obviously a real root.

Symbolic objects and corresponding commands for them work in an other way that is similar to known from algebra and trigonometry. For example, let us declare variable *x* and coefficients of a second-order polynomial *a*, *b* and *c* as symbolic objects by means of the command *syms*:

>> syms x a b c   d

Now, new symbolic objects may be constructed from these ones, for instance, a symbolic second-order polynomial:

*>> P=a*x^2+b*x+c*
P =
*a*x^2+b*x+c*

An other command may also introduce symbolic objects:

*>> Q=sym(' d1*x^3+a1*x^2+b1*x+c1 ')*
*Q = d1*x^3+a1*x^2+b1*x+c1*

In contrast to numeric mode, MATLAB does not require the above variables *x*, *a*, *b*, *c*, *P* and *Q* to have any particular numeric values. Now, some commands may perform their analytical transformations.

The command *expand* multiplies two polynomials introduced so far and gets 5th order polynomial *R* as the product:

```
>> R= expand(P*Q)
 R = a*x^5*d1+a*x^4*a1+a*x^3*b1+a*x^2*c1+b*x^4*d1+
       b*x^3*a1+b*x^2*b1+b*x*c1+c*d1*x^3+c*a1*x^2+c*b1*x+c*c1
```

However, this command has not been instructed to collect similar terms like you did in the school. The command *collect* can do this:

```
>> R1=collect(R)
 R1 = a*x^5*d1+(b*d1+a*a1)*x^4+(c*d1+b*a1+a*b1)*x^3+(c*a1
                                   +b*b1+a*c1)*x^2+(c*b1+b*c1)*x+c*c1
```

The last linear notation form is still difficult to recognize a polynomial. Try the command

```
>> pretty(R1)
```

and get more habitual view for the polynomial *R1*:

```
        5                    4                          3
   a x  d1 + (b d1 + a a1) x  + (c d1 + b a1 + a b1) x

                                 2
        + (c a1 + b b1 + a c1) x  + (c b1 + b c1) x + c c1
```

Students should understand that the latter hasn't been any object but simply a kind of typesetting on the screen.

Some MATLAB commands already introduced understand both numeric and symbolic objects; some commands may work with data of only one type.

Example 1.5. By means of symbolic calculation you may recall formulae for determinant:

```
>> A=[a  b; c  d]
>> det(A)
ans = a*d-b*c
```

Determinants of higher orders might be calculated for symbolic data as well. Try!

Example 1.6. Imagine you need to recall formulae for solving quadratic equation $P(x)=0$. Look:

```
>> roots=solve(P)
roots = [ 1/2/a*(-b+(b^2-4*a*c)^(1/2))]
        [ 1/2/a*(-b-(b^2-4*a*c)^(1/2))]
```

```
>> pretty(roots(1))
                              2         1/2
                      -b + (b  - 4 a c)
                 1/2 --------------------
                              a
```

Roots for the third order polynomial *Q*(*x*) may be also presented by algebraic formulae. Try!

Results obtained are valid for any coefficients. The user may wish however to obtain a polynomial for particular values of the coefficients. Substitution of those values may be realized in the following way:

```
>> a=1; b=1; c=1; P1=subs(P)
P1 = x^2+x+1
```

The object *P1* still remains symbolic one. Now, other commands that understand symbolic objects may work. For example, graphics of *P1*(*x*) may be plotted by the command *ezplot*(*P1*). (Note that the inverted commas has not been used because *P1* is symbolic with an undefined meaning of *x*). If required, any real or complex value of *x* may be substituted into the symbolic polynomial so that the latter will get a corresponded numeric value. For instance:

```
>> P1i=subs(P1, 1+i)          or
>> x=1+i; P1i=subs(P1)
```

produce the same numeric result

```
P1i =   2.0000 + 3.0000i
```

So, polynomials are treated in MATLAB either as numeric or as symbolic objects what gives a certain freedom to user. Besides, these objects may be converted to one another. The following command converts symbolic polynomial *P1* to its numeric analogue, i.e. to a vector of numeric coefficients:

```
>>P1num= sym2poly(P1)
P1num =     1     1     1
```

Roots of the former symbolic polynomial may be found now. In turn, numeric polynomial, i.e. its corresponded vector, may be converted into a symbolic one. Say, for the 4th order polynomial from the example 1.4 one gets

P4=poly2sym(p)
*P4 = x^4+2*x^3+3*x^2+4*x+5*

Some analytical transformations may be performed for it, for example its derivative may be found but this is the focus of the next section.

1.3.2. Symbolic mathematics in MATLAB

Due to symbolic objects, MATLAB became much clever so that it mastered the higher mathematic course of Ukrainian universities. Indeed, it can easily find derivatives or integrals of many functions:

Example 1.7. Find derivative and anti-derivative of $y = \frac{1}{1+x^2}$, plot their graphics, calculate the derivative at *x=2* and the area between the curve *y(x)*, axis *Ox* and lines *x=1* and *x=5*.

Solution. First, introduce symbolic variables *x* and function *y*,

```
>> syms x
>> y=1/(1+x^2);
```

Its graph

```
>> ezplot(y)
```

has been shown in the Fig. 1.5,A along with the area ABCD under the focus. Its derivative is

```
>> Der=diff(y)
Der = -2/(1+x^2)^2*x
```

Or, in more convenient form to us

```
>> pretty(Der)
              x
       -2 -----------
                2  2
            (1 + x  )
```

The value at *x=2* and the graph of the derivative are obtained by

```
>> Der2=subs(Der , 2)
Der2 =   -0.1600
```

The plot see now in the Fig. 1.5,B:

```
>> ezplot(Der)
```

Get the primitive function and its graph:

```
>> Int=int(y)
```

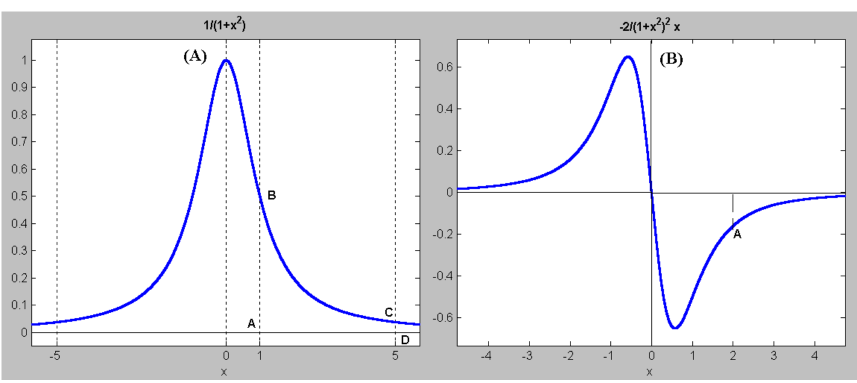


Fig. 1.5. The function and its derivative from the example 1.7.

Int =atan(x)
>> figure; ezplot(Int)
To find out the area restricted by ABCD one needs, as it is known from the higher mathematics, to specify limits of the integral:
>> Area=int(y, 1 ,5)
*Area = atan(5)-1/4*pi*
The latter expression still remains a symbolic one. Its numeric value may be either calculated
*>> atan(5)-1/4*pi*
ans = 0.5880
or evaluated by the command[8]
>> eval(Area)
ans = 0.5880

Further examples demonstrate other capabilities of MATLAB useful in student practice.

Example 1.8. Solve transcendental equation $\arcsin x = p(1-x)$ for $p = 4$.

Solution. Introduce symbolic variable and the function to find the root of
>> syms x p

[8] and the name of the command is clear from its role, to evaluate.

>> *F=asin*(*x*) – *p**(*1-x*)
and get particular form of the latter for $p = 4$:
>> *p=4*; *F4=subs*(*F*)
    F4 =asin(*x*) - *4+4*x*
Now, either command *solve* or *fzero* may be explored. Learn the first one:
>>*root=solve*(*F4*)
root =-sin(*-.89048708074438001001103173059554*)

The latter result looks out somewhat strange; indeed, why the value of *sin* has not been calculated? To answer, try the command *whos*: it returns the list of all variables already defined in current MATLAB session along with their class. It would be thus discovered that the variable root is a "sym object". To evaluate its numeric value, try:
>> *root=eval*(*root*)
root =   0.7774

Example 1.9. Solve transcendental equation $\cos x = .5x$
Solution. The command *fzero* (named after 'find zero') may be applied as well as the previous one:
>> *x=fzero*('*cos*(*x*) - *.5*x*' , *0*)
x =   1.0299
Note, that a second parameter, any real number for the *first guess*, is required by this command!

Example 1.10. Expand function $y = e^{-t^2}$ to Tailor series.
Solution. Functional series, particularly the Tailor series, are studied on the second study year. Despite of it, MATLAB can get the solution:

>> *syms t; T=taylor*(*exp*(- *t*^2), *13*)
 *T =1-t^2+1/2*t^4-1/6*t^6+1/24*t^8-1/120*t^10+1/720*t^12*
>> *pretty*(*T*)

$$1 - t^{2} + 1/2\, t^{4} - 1/6\, t^{6} + 1/24\, t^{8} - 1/120\, t^{10} + 1/720\, t^{12} \qquad (1.8)$$

Expansions like (1.8) will be studied in the Module 3.2.

It is natural that MATLAB contains a lot of commands to help students work with their higher mathematics. The command *det*, for example, servers for numeric evaluation of determinants of any order

(compare with the example 1.5). Command *inv* finds inverses of matrices.

It is natural that MATLAB contains ready functions for solving problems of linear algebra that students learn in their first year university course. (We would not only provoke students to use them instead of their "by-hand" solution. However, their use for checking by-hand solutions, in term papers and especially in diploma works is highly encouraged).

Example 1.11. Find inverse of matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1^2 & 2^2 & 3^2 \\ 1^3 & 2^3 & 3^3 \end{pmatrix}$.

Solution. Use a ready command for inverting matrices

```
>> A=[1 2 3; 1^2 2^2 3^2; 1^3 2^3 3^3];
>> A1=inv(A)
   A1 =   3.0000   -2.5000    0.5000
         -1.5000    2.0000   -0.5000
          0.3333   -0.5000    0.1667
```

Despite division of matrices like A/B is forbidden in mathematics, MATLAB uses *slash* / and *backslash* \ signs to denote commands for solving systems of linear algebraic equations (SLAEs). In fact, *A/B* denotes *A*inv*(*B*), but *A\B* denotes *inv*(*A*)**B* (dimensions of *A* and *B* should be kept correctly!). It means that solves the matrix *A\b* equation *Ax=b*. where *x* and *b* are correspondingly column-vectors of unknowns and of right hand side coefficients.

Example 1.12. Solve SLAE $\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1, \\ x_1 + 4x_2 + 9x_3 &= 2 \\ x_1 + 8x_2 + 27x_3 &= 3. \end{aligned}$.

Solution. As the matrix of the system has already been typed in MATLAB, one needs only the vector of its coefficients *b*=[*1*; *2*; *3*]. The solution is so

```
>> x=A\b
   x =  -0.5000
         1.0000
        -0.1667
```

Be advised to check the solution:

```
>> A*x
   ans =   1.0000
           2.0000
           3.0000
```

what is the given vector *b*. It would be useful for students to apply also other methods for SLAE, such as the Kramer's method and inverse matrix method.

More valuable examples solving typical mathematical problems might be found in [1,2,6].

Problems for Module 1

1.1. What of MATLAB interface windows did you explore? For what purpose are they useful for? How to call up Help in the MATLAB?

1.2. How to get help for plotting graphs? How to enquire the names of elementary functions realized in MATLAB?

1.3. For what do comments serve in MATLAB, how are they introduced?

1.4. What of the formats for real and complex numbers are ther in MATLAB? How to input vectors and two-dimensional arrays (matrices)?

1.5. How to construct legal names for variables in MATLAB? Examples of illegal names? Is it allowed to name variables *ans*, *pi*, *i*, *j*? How to assign numeric values to variables?

1.6. What of the numeric operations do you know in MATLAB? Are there any restrictions to operations +, –, *, / and ^ with regard to matrices? How do the above operations differ from similar ones "with the dot, i.e. " **.***, **./** and **.^**?

1.7. Is it possible to apply functions *sqrt*, *sin*, *exp* etc. to matrices? What would be the result? Is it allowed the complex numbers to be arguments of the above functions? Could you compare this with other programming languages?

1.8. How to separate commands in the MATLAB's Command Line? What is the difference between separators "**,**" (comma) and "**;**" (semicolon)? How to repeat a command executed earlier in the current session?

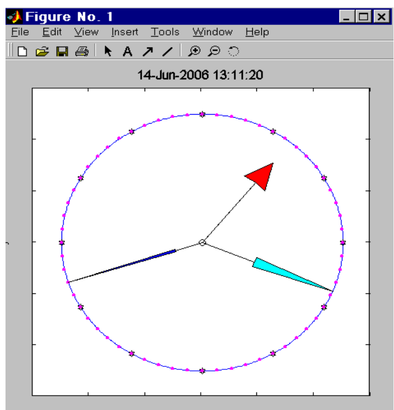


Fig.1.6. Program *MyClock* developed in MATLAB (see program in Attachment *A3*)

1.9. What do the commands *length*, *size*, *inv*, ' and *diag*? Do you know other commands that operate with matrices?

1.10. For a given vector, shift its elements for one in counter clockwise direction. (Hint: remember construction (1.4)).

1.11. What mean the command like *x=-pi* **:** *pi/100* **:** *3*pi* ? How is it used for plotting graphs?

1.12. What of commands for plotting graphs do you know? What's the difference between the commands *plot*, *fplot*, *ezplot*, *comet*?

1.13. How may the commands *legend*, *title*, *grid*, *xlabel*, *ylabel*, *axis*, *insert* "decorate" your graphs? How could you set or change color of your plot curves?

1.14. For what do one use the command *figure*? If you need to plot several graphics, how could you plot some of then in one window but other curves in an other?

1.15. Plot the function given parametrically

$$x = \sin(3t)\cos t, \quad y = \sin(3t)\sin t, \quad t \in [0, \pi] \quad (1.9)$$

1.16. How would it be possible to plot a discontinuous function like in the example 2.2 or continuous piece-wise function in problems 2.12, 2.13?

1.17. How would you explain the difference between *numeric*, *text* and *symbolic data* in MATLAB?

1.18. For a given matrix *A* find the mean of its elements.

1.19. How to find roots of a polynomial? How many are them? How to find derivative and primitive of a given function? How to find a define integral?

1.20. Develop a MATLAB program that displays current time like the one shown in the Fig. 1.6. (Hint: use commands *date* and *clock*).

Module 2: Basics of MATLAB programming

General module characteristics: *Programming*, i.e. making *computer programs* is useful to automate calculations that you would repeat many times otherwise, or to make calculations without manual intrusion. Commands suggested by structural programming paradigm will be introduced, examples of simple but motivating to further work programs will be given. Flow charts are used to explain programs.

Module structure

Micromodule 2.1. *m*-scripts and *m*-functions
- 2.1.1. *Scripts*, the simplest programs
- 2.1.2. MATLAB' Functions (*m-function*s)
- 2.1.3. Difference between Scripts and Functions

Micromodule 2.2. Structured programming in MATLAB
- 2.2.1. Loop operator *for ... end*,
- 2.2.2. logical operator *if ... else ... end*
- 2.2.3. logical arithmetic with *and*. *or*, *not*

Micromodule 2.3. More MATLAB' programs
- 2.3.1. Periodic Step-function
- 2.3.2. Least element of an array
- 2.3.3. Re-ordering of a vector

Micromodule 2.4. Supplementary problems
- 2.4.1. Dialogue programs
- 2.4.2. Debugging programs

Problems for Module 2

Micromodule 2.1. *m*-scripts and *m*-functions

To automate multiple repetitions of the same commands, one writes *programs*. Actually, all the commands you learned previously like *fplot* are the programs written by somebody. Now, let's become programmer, too!

2.1.1. *Scripts*, the simplest programs

Assume, you need to systematically create triplets of functions, say *F1*, *F2* and *F3*, and then compare them by means of plotting their graphs. These functions, each in the form of two arrays $x=\{ x_1, x_2, \dots x_n\}$ and $F1=\{ F1_1, F1_2, \dots F1_n\}$, may be created "by hands" like you did in

the Module 1.2. However, it would be a good idea to collect all the commands required for plotting all three functions, i. e. their visualization, and make the MATLAB to execute them by a single command. Well, let us prepare a special file and save it under the name *visualize.m* (with extension *.m*!). The text content of such a program saved in a file is usually called *its listing*.

The following listing of the program *visualize.m* is suggested:

Listing 2.1 of the file "*visualize.m*":

```
1  % Example of a simple Script Program
2  % to visualize and compare graphics of three functions
3  % named F1, F2 and F3
4  % of argument x created previously in Workspace
5
6  % Copyright Ye. Gayev
7
8  figure; plot(x1,F1,'r',  x2,F2,'b',  x3,F3,'g')
9  title('Comparison of three Functions')
10  grid on;
11 xlabel('Independent argument X'); ylabel('Functions F of X');
12 legend('F1', 'F2', 'F3')
13 % end of the script
```

Do the following to create this file practically: a) Press the icon ("New *m*-file") in the MATLAB; "*m-File Editor*" appears with an empty window. b) Type above sentences line by line in this window. Do not type numbers as the numbering will appear automatically. c) Having typed all the text, press to save the file. Type the name *visualize* instead of the name *unnamed* suggested. The file will be saved with the extension ***.****m* in the subdirectory *WORK*. Before using it, let us analyze its content.

Lines 1 to 6 start with the sign *per cent %* as well as the line 13. This symbol denotes *comments* which the program does not account for. The use of comments is explained below. Empty lines like 5 and 7 may be used by programmer to better emphasize the structure of the program and make it easily readable.

Section with MATLAB commands follow the comments. Commands in the line 8 call new graphical window and plot all three functions in one window with different colors, red, blue and green. The command 9 prints the title in the window. Command 10 draws grids, and the commands 11 label horizontal and vertical axes. Finally, the command 12 provides legends which mean that the red curve corresponds to *F1*, and so on. The line 13 is optional and serves for denoting the end of the program.

Now, make few preparatory calculations and call the new progam:

```
>> % Create argument array
>> x1=-1 : .01 : 3; x2=x1; x3=x1;
>> % Create arrays with three functions
>> F1=x1; F2=F1 .*x1/2; F3=F2 .*x1/3;
>> % Finally, get result and analyze it!
>> visualize
```

A graph with three curves will appear. Problems 2.1 and 2.2 suggest some exercises. The program *visualize* developed becomes a command of MATLAB now that will be used several times in this book.

2.1.2. MATLAB' Functions (*m-functions*)

Another kind of MATLAB program is called *m-functions*. Let us consider an example of a program that makes *numerical differentiation*. Imagine, one has a number of function values $y=\{y_1, y_2, y_3, \ldots, y_N\}$ that correspond to arguments $x=\{x_1, x_2, x_3, \ldots, x_N\}$. One may interest in getting knowledge how fast the function changes with the argument. In the case of analytically given function $y(x)$, its derivative $y'(x)$ would answer the question. We deal with a *table function*, so an approximate formulae should be used[9]:

$$p_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}, \quad i = 0,1,\ldots,N-1.$$

The program we would like to have should get arrays x and y and return arrays *y1* with derivatives and *x1* with corresponding arguments (note

[9] This formula uses "*Differences Up*". See any course of numerical mathematics such [4] for alternative formulas.

that their lengths are less for 1, i.e. $N-1$). The following program *MyDiff* solves the problem.

Listing 2.2[10] of "*MyDiff.m*"

```
1 function [P,X1]=MyDiff(X,Y)
2 % This program returns Derivative of the function Y=Y(X)
3 % Copyright Ye.Gayev, July 2006
4
5  X1=X(1 : end-1); % Coordinates of the derivative P(x);
6  X2=X(2 : end);  dX=X2-X1;
7  Y1=Y(1 : end-1);  Y2=Y(2 : end);  dY=Y2-Y1;
8  P=dY./dX;  % Formulae "Differences Up" is used
9 % End of differentiation
```

The program consists of 9 rows to be typed via the *m*-File Editor and saved in a file with the name *MyDiff.m*. As before, lines 2 to 3 form section with comments. Besides, comments serve for explanation and providing additional information in lines 5 and 8. Note however that the first line of any *m*-function should obligatory be the declaration *function* along with definition that output variable *OutVariable* is linked with input variable *InputVariable* through the function name *FuncName*. It is declared by means of simple *syntax*

$$OutVariable = FuncName(InputVariables) \quad .$$

In our case *OutVariable* is an array [*P*, *X1*] with variables *P* and *X1*, and *InputVariables* are given arrays *X* and *Y*. The *FuncName* is *MyDiff*.

The command 6 creates an auxiliary array *X2* with the data from *X* but shifted for 1. Then, array *dX* is created in the line 6 with differences between neighboring arguments. Similar, differences of function values are stored in the array *dY* created on the line 7. Finally, $N-1$ values of the derivative is obtained by the command 8.

Note that presence of *input* and *output* arguments contrasts the *m*-function with *m*-scripts where no arguments are used at all. Below is an example of using the new program.

1. First, try asking help for the new program:

---

[10] See the same program in the Attachment A1 in an advanced version that analyzes "quality" of input and uses *switch ...end* statement. See also program *MyDeriv*, listing 2.10.

```
>> help MyDiff
 This program returns Derivative of the function Y=Y(X)
 Copyright Ye.Gayev, July 2006
```

It is to conclude: The Comment Section in each program serves for getting help; it is worth to provide information in it about purpose of this program, how to use it and other relevant information (copyrighted person for example). Pay attention that it is very important to document your program clear and completely! Another example see in Listing 2.4.

2. To work with the new program, let us get "experimental" data by the following way (any function might be used instead of $\sin t$):

```
>> t=0 : 2*pi/10 : 4*pi ;  y=sin(t) ;
```

(Note, the function data are rather rough because of a big increment $dt=\frac{\pi}{10}$). Differentiate these data numerically:

```
>> % Examination of MyDiff program
>> [F1,x1]=MyDiff(t, y);
```

Now, get "experimental" data 100 times more precisely and differentiate them again:

```
>> t=0 : 2*pi/1000 : 4*pi ;   y=sin(t) ;
>> [F2, x2]=MyDiff(t, y);
```

It is worth to compare both results with original function $y = \sin(t)$ in the same Figure. First, one needs to rename *t* and *y* as *x3* and *F3*, and then visualize all three curves:

```
>> x3=t; F3=y;
>> visualize
```

Results have been shown in the Figure 2.1 and bring the following conclusion: numeric function *F1*(*x*) is rather depart from the function *F2*(*x*) which, in contrast, almost coincides with the exact derivative $y = \cos t$ of the given function. Students are advised to make "computer experiments" by varying increment *dt* and even with other functions, as suggested in Problems 2.1.3 to 2.1.6.

2.1.3. Difference between Scripts and Functions

It is important to account for several significant differences between *scripts* and *m-functions*:

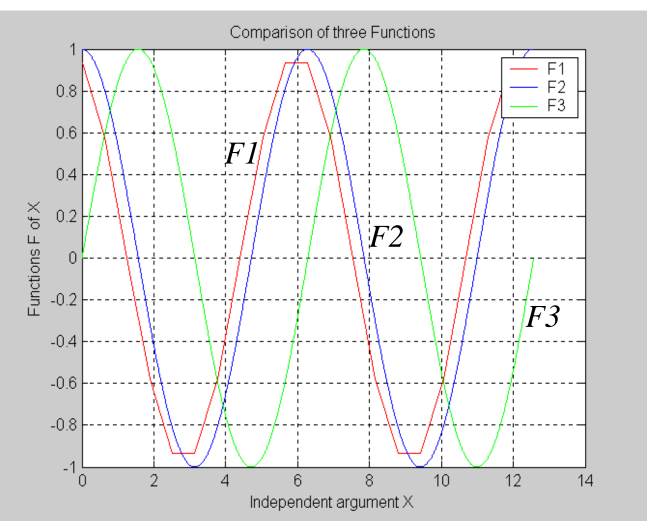


Figure 2.1. Results of a "computer experiment" with numerical differentiation by the program *MyDiff*: *F3* is original, but *F1* is its rough and *F2* is more precise derivatives.

(*i*) Scripts do not have neither input parameter, nor output parameters in contrast to *m*-functions that have them by definition.
(*ii*) Because of this, scripts may use any variables already defined in MATLAB environment. In contrast, *m*-functions cannot "see" any variables from the environment except those listed in *InputVariables* or declared as *global* (see below). Their relation to the MATLAB environment has graphically been illustrated in Fig. 2.2.
(*iii*) Similar to this, any variables created within scripts may later be used in the environment (by a next script or *m*-function, for example). In contrast, variables created within *m*-functions may be used within it but are not "seen" from outside (unless they declared as *global*). It is convenient: if you introduce any identity *NewVar* within function, you do not need to check if it was defined somewhere before. From another hand, when you leave *m*-function, the program immediately "forgets" all variables created within it.
(*iv*) *m*-functions may contain internal (nested) *m*-functions that are called *subprograms* or *sub-functions* (an example in Listing 2.7). Similar to (*iii*), variable created in a nested function is not seen in the external one.

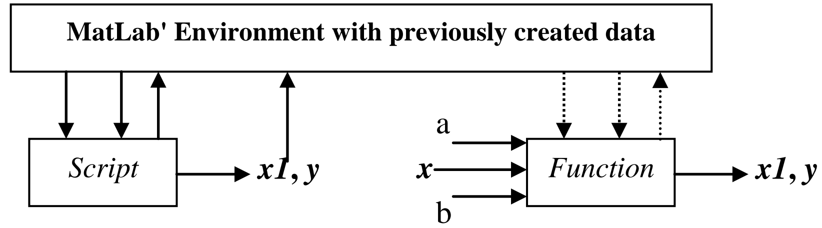


Fig. 2.2. Relations of ***scripts*** and ***functions*** with the whole MATLAB environment: solid lines denote direct exchange of data while dotted ones exchange by *global* variables.

The "legal" way to supply variables to *m*-functions lies through input variables. Input *t,y* and output variables *F1,x1* in the above example program *MyDiff.m* are called *formal parameters* of the program. They are substituted by other *real parameters* when *MyDiff* is called.

This way may however be insufficient. Another one is declaration of variables to be global, *global NewVar1 NewVar2* . This should be declared before the variable is created, both within the function and outside it, or both in nested and external function. Be advised to check visibility and values of such variables during debugging your complex programs (see Micromodule 2.4.2)!

Micromodule 2.2. Structured programming in MATLAB

Computer programs given over might be called *linear programs* as they are executed line-by-line in a top-down manner. Your programs will be more "intellectual" if you apply logical operators in them. There was a command *GOTO* in first algorithmic languages that could change *flow of program* with respect to a logical condition. However, if programs become sufficiently complex and hierarchical, they are extremely difficult for reading and understanding them. It was so suggested by N. Wirt and E.V. Daikstra (pronounce Дейкстра) not to use the *GOTO* statement but use specially defined programming blocks, *control-flow statements*, instead. Such kind of work was called the *structural programming*[11]. MATLAB's programming implements most decisions of contemporary computer science. *Flow charts* may significantly facilitate understanding of a complex program.

[11] For details see http://en.wikipedia.org/wiki/Structured_programming.

2.2.1. Loop operator *for … end*

This composite operator allows repetition (*loop*) of several statements a specified number of times. Its general *syntax* has the form:

for *LoopVariable=Value1* **:** *Increment* **:** *Value2*
a statement or a command 1;
. . . . .
a statement or a command K;
end

This block of commands works in the following way illustrated by the flow chart in the Fig. 2.3. First, the variable *LoopVariable* is set to the initial value *Value1*. Condition if *LoopVariable* > *Lalue2* is checked, and statements and commands from *1* to *K* are executed because the answer is *false* (*No*). Having reached the *logical bracket* ***end***, the program repeats execution of all the statements and commands from *1* to *K* again but with *LoopVariable=Value1* + *Increment*. Next execution uses *LoopVariable=Value1* + 2**Increment*, and so on until *LoopVariable* becomes more than the value *Value2*. In this case, the answer to the above question is *true* (*Yes*), and the program block ends its work.

Comments: 1. The *Increment* may not be obviously positive; in the case of negative *Increment*, the value of *LoopVariable* each time reduces. 2. The loop described may be prematurely terminated by the statement *break*, see *Help break*. 3. Each of statements may use the operator *for ...end* again. Such new loop would be called the *nested loop*. Let's learn few examples.

Example 2.1. Rotation of a stick on your PC screen.

You easily can, of course, plot a static stick with coordinates $(\cos t, \sin t)$ and $(-\cos t, -\sin t)$ on your PC screen for any given value t. For example:

```
>> t=pi/4 ;
>> x1=cos(t) ; y1=sin(t) ;
>> x2=-x1; y2=-y1;
>> plot([x1   x2], [y1   y2], 'b', [0], [0], 'or')
```

The straight line you see on the screen is symmetrical against centre $(0,0)$ that is emphasized by red circle. The program *helicopter* uses the *for ...end* operator to rotate this stick on your screen:

Listing 2.3 of the script "*helicopter.m*"

```
%Script HELICOPTER
% produces a stock that revolves N times
% in counter-clockwise direction.
% Copyright Ye.Gayev, May 22, 2005

 Dt=.1*pi;   N=10;
for t=0 : Dt : 2*N*pi
        x1=cos(t) ; y1=sin(t) ;
        x2= -x1; y2= -y1;
        Pl=plot([x1, x2],  [y1, y2], 'r') ;
        set(Pl,  'linewidth',4 ) %See Help or  Module 4 for explanation of set !
        axis([-1.1  1.1  -1.1  1.1]) % Why do we use this?
        hold on; plot([0], [0],'o'); hold off;
        pause(.1) % See Help for pause
end
% End of helicopter
```

Comments: 1. Mistakes in program may lead to getting caught in an endless loop. Use keys <*Ctrl* + *C*> to break loop and stop the program. 2. More sophisticated operators *while ... end* and *switch ... end* could also be very useful in practice; see Help for them. Few more examples of *for-loop* have been given in listings 2.7, 2.9 and A1 (*MyDiff*).

2.2.2. Logical operator *if … else … end*

This command makes your programs more "clever". Its simplest syntax looks as

```
if  LogicalExpression
    Statements1
else
    Statements2
end
```

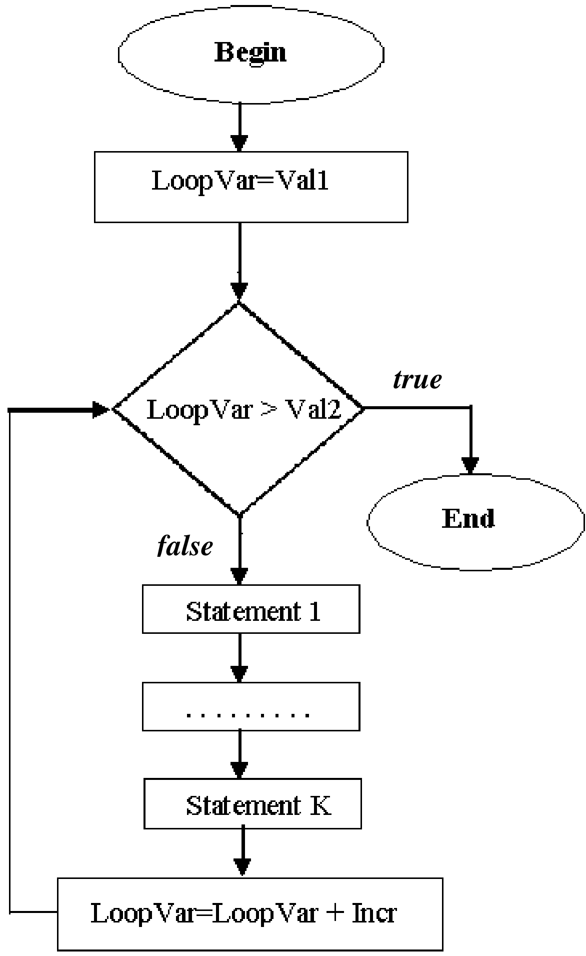


Fig. 2.3. Flow chart of the *for … end* statement

This command block is self evident, so we may explain it by an example.

Example 2.2. Make MATLAB to understand the following mathematical piece-wise *Step-function* (that is associated with famous Heaviside and Dirac functions)

$$y(x) = \begin{cases} +1, & if \quad x > 0 \\ -1, & if \quad x \le 0 \end{cases}$$

+1

-1

It means, MATLAB should calculate value of the Step function, i.e. +1 or -1, for each (at least single) argument x and plot its graphics shown on the right hand-side. The program *Step01* solves this problem.

Listing 2.4 of "Step01.m"

```
function y=step01(x)
% Program-function to plot piece-wise function
%       /  +1,   if   x > 0
%  y=|
%       \  -1,   if   x <= 0
% Example of use: ezplot('step01(x)', [-3 3]).

% Copyright Ye. Gayev, Nov. 2005

if      x<0     % Note: this is a  Logical Expression !
        y=1;    % Here is Statement1
else
        y=-1;   % Here is Statement2
end
% End of Step01
```

Let us explain now how this program works:

1. If one asks MATLAB for a help,

>> help Step01

the answer is

```
Program-function to plot piece-wise function
     /  +1,   if   x > 0
  y=|
     \  -1,   if   x <= 0
Example of use: ezplot('step01(x)', [-3 3]).
```

So, the program uses the information provided in the Comment Section to remind to user how this function looks like and how to use it. We may follow to the last advice.

2. Try to calculate this function for several argument values, for instance:

```
>> y1=step01(2), y2=step01(-2)
    y1 =    1
    y2 =  -1
```

So the program calculates correct.

3. Now, plot the graph of the Step-function in the way recommended:

```
>> fplot('step01(x)', [-2 2 -1.2 1.2])
```

Graphics similar to the above picture is to be displayed. Congratulation: MATLAB knows now the new mathematical function you've created!

Pay attention however that the function works wrongly for array arguments. Try for example and get a strange answer:

```
>> x=[-2   -1   1   2];   y=step01(x)
    y =     1
```

Our next aim will so be to teach our program to understand arrays. For this, consider problems 2.11 – 2.13 and learn Micromodule 2.2.3. At the moment, however, consider a new useful example.

Example 2.3. Develop a program *welcome* that will greet you with regard to the day or night time moment and remind you to go sleeping when it is too late (take control moments 6 a.m. and 1, 6 p.m. and 0 a.m.). The program should be of course of the script type:

Listing 2.5 of "*welcome.m*"

```
% Script WELCOME
% greets the User depended on the part of Day or Night
% divided into 6 hours, 13, 18, 24 and [0, 6] hours

% Copyright of Ye. Gayev, Dec. 2005

T=clock;   %Returns the array [Year, Month, Day, Hours, Minutes, Seconds]

if   (T(4)>5) & (T(4)<=12)
        disp('   ') % to get an empty string on the screen
        disp('Good morning!')
elseif   (T(4)>12) & (T(4)<= 17)
        disp('   ')
        disp('Good afternoon!')
else
```

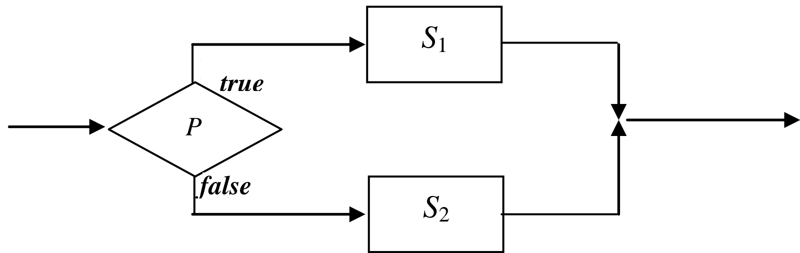


Fig. 2.4. Flow chart explaining logical operator *if P ... else* S_2.

```
    if (T(4)<= 23)
          disp('   ')
          disp('Good night!')
    else
          disp('   ')
          disp('Good night. But I wish you slept this time!')
    end
end
```

Explanation. First, the program uses two MATLAB commands unknown to you, *clock* and *disp*. Ask Help about them or/and try them from the Command Line. You will be answered with concern to *clock* that it returns a numeric vector with six elements that corresponds to date and time moment in the format noticed in the comment of the 7th line. We need to work with the only 4th element *T*(*4*).

Secondly, the *IF*-operator we study is applied in its general form here,

```
if  LogicalExpression1
    Statements1
elseif  LogicalExpression2
    Statements2
else
    Statements3
end
```

The latter works in the following way. Having reached the *if*-operator, computer checks the *LogicalExpression1*. If it has the value *true*[12],

[12] See next section 2.2.3.

several *Statements1* are executed after which the program leaves the *if ... end* block at all. However, if the *LogicalExpression1* is *false*, the portion of *if*-block between *elseif* and *end* becomes working. Again, *LogicalExpression2* is checked. If it is *true*, *Statements2* are executed and the program goes outside the *end*. However, if *LogicalExpression2* equals *false*, the S*tatements2* are skipped and *Statements3* are executed after which the program comes to the line next after *end*. See also illustration in the Fig. 2.4.

First two groups of *Statements* include two commands. It is important to note that *Statements3* contain one operator that is a nested operator *if…else…end*. The latter works as explained over. Another point to draw attention is the manner of writing all the above programs: they look graphically out like steps. Students are advised to always write your programs in such a way to emphasise blocks and other structural elements of the program. MATLAB' *m*-file Editor makes this automatically what evidences a "good taste" of this programmer!

2.2.3. Logical arithmetic with *and*, *or*, *not*

In the listing 2.5, we met complex *logical expressions* like

$$(T(4)>5)\ \&\ (T(4)<=12)$$

that consists of two elementary expressions $T(4)>5$ ("*Element T*(*4*) *is more than 5*") and $T(4)<=12$ ("*Element T*(*4*) *is less or equal than 12*"). What is the *logical expression*?

It is a sentence that may be estimated in terms of *true* or *false*, i.e. may have values of 1 or 0 only, and depends on the variable argument. (Sentence itself does not depend on any argument; for instance, sentence "*The sun rises in the West*" is always *false*). Say, in 3 o'clock the expression $T(4)>5$ gets the value 0 (*false*) while in 7 a.m. its value is 1 (*true*). *Elementary logical expressions* may include relation signs as

= = (*equal*[13])	< (less)	> (more)
<= (less or equal)	>= (more or equal)	~= (not equal).

Like in every-day-life, we can formulate complex expressions from elementary ones. *Binary* logical operations

& (AND) and **|** (OR)

and *unary* logical operation

[13] Note that one assignment sign = cannot be used for relations!

~ (NOT)

are used for doing this. Logical value of complex expressions may often be estimated by intuition; for example, the expression

$$(T(4)>5)\ |\ (T(4)<=12)$$

equals *true* both in 3 and in 7 o'clock. However, computers require rigorous formal rules. The latter, known as logical arithmetic, have been given in tables below.

	A&B	
	A=true	*A=false*
B=true	1	0
B=false	0	0

A | B	
A=true	*A=false*
1	1
1	0

	~ A
A=true	*false*
A=false	*true*

Most of algorithmic languages do not mix logical and arithmetic operations. In contrast, MATLAB allows mixing of both: a number may multiply logical expression *A*, and the result is either the same number if *A*=1 (*true*),or zero if *A*=0 (*false*). This MATLAB' feature lead to simplification of many programs.

Recall the programs *Step01* for plotting discontinues function. Here is its modification.

Example 2.2,A: Step-function programmed with MATLAB' logical feature.

Listing 2.6 of "*step02.m*"

```
function y=step02(x)
% Program-function to plot piece-wise function
% using features of the MATLAB Logic
%       /  +1,   if x > 0
%  y=|
%       \  -1,   if x <= 0
%
% Example of use: ezplot('step01(x)', [-3   3]).
% The command plot may be used as well.

% Copyright Ye. Gayev, Nov. 2005
y1=(x<0);
```

```
y2=(x>=0);
y=-y1+y2;
% End of "step02.m"
```

It is easy to check that all the easy-to-use plotting programs do work with this command, for instance:

```
>> fplot( 'step02(x)' , [-3   3   -1.1   1.1])
```

Check also that the program is applicable to array data:

```
>> y=step02([-2 -1 0 1 2])
     y =    -1    -1     1     1     1
```

It is no wonder so that the command *plot* does work too:

```
>> x=-4 : 8 / 1000 : 4 ;   y=step02(x) ;
>> plot( x , y),   axis([-4   4   -1.1   1.1])
```

We have thus taught MATLAB to understand array arguments what is in compliance with the MATLAB's "matrix philosophy". Compare this function *step02* with another one *step03* suggested by the problem 2.11.

Micromodule 2.3. More MATLAB' programs

Three more programs placed below have to enhance your programming skills and are thus useful for further applications in other disciplines.

2.3.1. Periodic Step-function

An important role in mathematics, physics and informatics plays periodical modification of the Step-function in Example 2.2, with period 2π for example:

$$y(x)=\begin{cases} +1, & if \quad x\in(-\pi+2k\pi, 2k\pi] \\ \\ -1, & if \quad x\in(2k\pi, 2k\pi+\pi] \end{cases}$$

$k=0,1,2,...$

Its graphics recalls the function $\sin x$ but is discontinues. How to define this function in MATLAB, how to plot its graphics? Program *Step_pi* has been suggested here.

The following algorithm is suggested for determining $y(x)$ for arbitrary x. First of all, check if $x\in[-\pi,\pi]$. If *Yes*, any previously developed function *Step01*, *Step02* or *Step03* may be used to assign to

y the value either -1 or $+1$. We would suggest this to be a nested subfunction *step*, see Listing 2.7.

If the answer is *No*, let us shift along axis *Ox* periodically for 2π to the right (or to the left) until x will be captured into the interval $[-\pi + k \cdot 2\pi, \pi + k \cdot 2\pi]$ of the length 2π where $k = +1,+2,+3,...$ (or $k = -1,-2,-3,...$). It is not clear in advance how many steps are to be done (and thus the *while* –loop is to be used), but it is evident that the process will be finite. When this happens, the same function *step* may be used. Such shifting is equivalent to subtractions $x = x - 2\pi$ if $x > 0$ and to additions $x = x + 2\pi$ if $x < 0$. This algorithm is schematized in the flow chart in Fig. 2.5. Listing 2.7 realizes the algorithm for MATLAB.

Listing 2.7 of *step_pi.m*

```
function  F=step_pi(x)
% Periodic Step Function,
% i.e. 2*pi-periodical continuation of the Step Function
%          / =-1;    if  x \in [-pi, 0]
%  F(x)=|
%          \ = 1,    if  x \in (0, pi]

% Copyright Gayev Ye.A., January 2006
L=length(x);  % See HELP for this function
for i=1 : L  %  Begin of loop № 1 on vector elements
    xx=x(i);
    if  (xx >= -pi) & (xx < pi)  %  If xx \in [-pi,  pi]
      %  disp('x within [-pi, pi]');
        F(i)=step(x(i));
    elseif   xx <= 0 %  If xx out of [-pi,  pi] and negative
        % disp(' X < 0 ' );
         nT=0;
        while ~ ((xx >= -pi) & (xx < pi))
        %Begin of loop № 2 until xx \in [-pi,  pi]
                nT=nT+1;   xx=xx+2*pi ;
        end
```

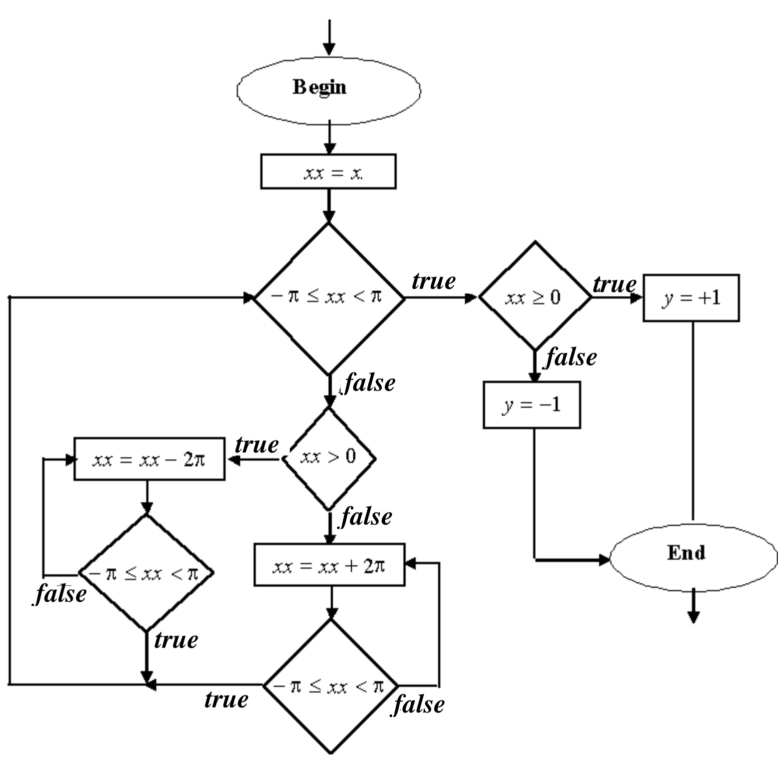


Fig. 2.5. Flow chart of periodical function *Step_pi*

```
  % End of loop № 2
   F(i)=step(xx);
  else              % If xx out of [-pi,  pi] and positive
     % disp(' X > 0 ');
          nT=0;
   while ~((xx >= -pi) &(xx < pi ))
        %Begin of loop № 3 until xx \in [-pi,  pi]
         nT=nT+1;   xx=xx-2*pi;
   end
       % End of loop № 3
  F(i)=step(xx);
 end  % Закінчення перевірки положення xx відносно [-pi, pi]
end          % End of loop № 1
```

```
% End of the program step_pi

%---------------------------------------------------------
%   Example of (nested) SubFunction
%---------------------------------------------------------
function  F=step(x)
% Step Function equals -1 if x \in [-pi, 0] but  = 1, if x \in (0, pi]
   if x<=0
      %   disp( ' SubProgram X<0' )
      F=-1;
   else
      %   disp( ' SubProgram X>0' )
      F=1;
   end   % End of SubFunction
```

It may be checked that the program *step_pi* works for array argument.

Comments to program. 1. Program *length*(*x*) determines the number of elements of *x*. 2. Command *disp*(*' Text '*) displays the text *Text* on computer screen. Such commands are useful during debugging of programs and checking if it follows the logic you incorporated. They may be either deleted or commented by *%* after tuning the program.

2.3.2. Least element of an array

An one-dimensional array *X* is given consisted of numeric numbers. Develop a program *MyMin* that returns the least element of them along with its position in the array.

Algorithm of the program is clear from MATLAB program in the listing below. It is assumed initially that *Xmin* is the first element of *X* and its number is *Imin=1*.

Listing 2.8 of *MyMin.m*

```
function [Xmin, Imin]=MyMin(X)
%Program for determination of smallest element of the vector X
% for teaching purposes
L=length(X);
Xmin=X(1);   Imin=1;
for i=2:L
        if  X(i) < Xmin
              Xmin=X(i); Imin=i;
        end
end
```

Similar to this program, a program *MyMax* may be developed that looks for greatest element of input array and its position. It is assumed below, in the next section, that this program exists. In fact, such programs *min* and *max* have already been included as built-in functions of MATLAB. It would be of interest to compare whose function is better, see Micromodule 3.4.

2.3.3. Re-ordering of a vector

Reordering elements of vectors is a key problem of computer science: for given vector *X*, return vector *Y* with the same elements ordered in ascending or descending order. The program *MyOrder* below analyzes one of arguments to be either *Increase* or *Decrease*, and warns user to correct the task otherwise. The following algorithm is used for the case *Decrease* for example. The problem is solved by *L=length*(*X*) steps. Each time *i=1, 2, ..., L* an auxiliary array *X1* with elements from *X* is checked, its greatest element by *MyMax* is found, placed to the *i*-th place of another auxiliary array *y* and new array *X1* is created with those element absent. Try to draw flow chart of the algorithm yourselves (problem 2.15.). The latter is realized in the following program.

<u>Listing 2.9 of *MyOrder.m*</u>

```
function Y=MyOrder(X, Order)
% Return vector Y with the elements of the argument vector X
%                                 / increasing elements if  'Order' is 'Increase'
% but ordered accordingly to |
%                                 \  decreasing elements if  'Order' is 'Decrease'
%
% Example: Y=MyOrder([1 2 3], 'Decrease') returns Y=[3 2 1].
% Uses MyMax and MyMin already developed

% Copyright Ye.Gayev, Dec. 2005

L=length(X);  X1=X;
switch Order
     case {'Increase', 'increase'}  % Rearrange X in Increasing Order
          for i=1 : L
               [Y1,I]= MyMin (X1);
               y(i)=Y1;  X1=[X1(1 : I-1),  X1(I+1 : end)];
          end
          Y=y;
```

```
case {'Decrease', 'decrease'} % Rearrange X in Decreasing Order
    for i=1 : L
        [Y1, I]=MyMax(X1);
        y(i)=Y1;  X1=[X1(1 : I-1),  X1(I+1 : end)];
    end
      Y=y;
otherwise
    disp('  ')
    disp('It should be either "Increase" or "Decrease" in the second entry.')
    disp('Check your problem and try again, please!')
    disp('  ')
end
% End of the Program
```

The program in the above example works irrelevantly on letter case, either *'Decrease'* or *'decrease'*. Such programs that analyze input not to include errors and warn user if so, create am impression of being intellectual. The program works with only row-vectors, then. It could easily be generalized however for matrices, as in below program:

Listing 2.9A of *My1Order.m*

```
function Y=My1Order(X, Order)
% Function returns matrix Y
%with the same dimensions as matrix X
%but elements of each row re-ordered as stated by Order.
% "MyOrder.m" is used as sub-program.
% Copyright Ye. Gayev. October 2006.

[N,M]=size(X); %N=number of rows in X
for i=1:N
   Y(i, : )=MyOrder(X(i, : ),Order);
end
```

Algorithm realized in the programs is one among several known algorithms of sorting, see [8,9] and problem 2.16. Because of importance of such algorithms in computer science, similar function *sort* was developed in MATLAB. A question will be addressed in Micromodule 3.4 which of algorithms "is better".

Micromodule 2.4. Supplementary problems

2.4.1. Dialogue programs

It is expected that computers will understand oral commands and communicate with us by voice in the future. Making computers so intellectual will also be up to you when you become specialist. Now, we shall make a first step and teach computer to communicate in a form of *dialogue from the Command Line*. Next step, communication with computer via Graphical User Interface, will be done in the Module 4.

Three ready MATLAB commands are sufficient to us here. The command *disp*(*x*) displays value of variable *x*, i.e. matrix in general case, without printing the name of variable; the command *disp*(*' Text '*) displays the text provided in inverted commas. The command

X=input(' *PromptText* ')

prints the text *PromptText* on the PC screen, waits for any input from the Command Line and assigns the latter to the variable *X*. Finally, the command *pause* (in its simplest syntax) stops the program until any key is touched.

Example 2.4.1. Develop a program *MyDeriv* that greets you, introduces itself and asks for which mathematical function would you like to get derivative function, and finally compares graphics of both origin and derivative. The following program realizes one of possible solutions.

Listing 2.10 of *MyDeriv.m*

```
% Script of a Dialogue Program
% that plots function provided by User
% and compares it with its derivative

% Copyright Ye.Gayev, June 2005

welcome;
disp('   ');  % to make space between lines
disp('  Here is Command Line Dialogue Program ');  %Commands to dialogue
disp('for analytical calculation of derivative ');
disp(' of a function you will be asked for.');

syms Fname x % List of variables to assign symbolic values

disp('   ');
disp('So, please input a Function of x which Derivative')
disp('    you would like to compare with. ')
```

```
Fname=input(' '); % Introducing function to differentiate

Deriv=diff( Fname );

disp(' You introduced function y=');
pretty( Fname )
disp(' Its derivative is y=');
pretty( Deriv )
disp('   ');
disp('Press any key and compare them in two graphics!')
pause

figure; ezplot(Fname);  title( 'Plot of your function')
figure;  ezplot( Deriv );  title(' Plot of your derivative')
disp('   ');
disp('Thanks for using this program!')
% End of MyDeriv
```

Please analyze yourself how this program works bearing also in mind comments in it. In developing programs of dialogue types, you are advised to do all your best for making your program polite and clear, i.e. sufficiently documented, for users.

2.4.2. Debugging programs

There is a saying among experienced programmers "No program may appears at first without errors". Errors may be occasional mistypes or even logical errors in programming. That is why you are recommended to draw flow charts for each complex program. Anyway, *debugging* is required to check each new program in all possible situations.

If your program contains mistypes or *syntax errors*, MATLAB produces a message colored red like

"*??? Undefined function or variable '. . .'*", or

"*??? Error: File: C:\MATLAB6p5\work\MyDeriv.m*

Line: 22 Column: 19 ")" expected, ";" found".

Find error in program by *m*-file Editor and correct it.

The most insidious and hidden are logical errors, however. To prevent them you are advised to split each complex program to blocks or sub-programs and check them. Being sure in parts, check your program as a whole. You shall often come back to program text in *m*-file Editor. It may be useful to find critical points of the program and

place control prints like *disp*('*I am at point A*') there to check if the program passes them properly. Having tuned the program, delete those checking points or, rather, comment them by *%*.

In fact, this technique realized in the *m*-file Editor so that you may do the same through menus *Debugging* and *Stop Points*. Try this! This service found further development in versions 7.1 till 7.3 of the MATLAB.

It may be thought that MATLAB is an *interpreter* rather than *compiler* what means that it cannot produce autonomous executive files. In fact, it was true for its versions less than v.5. In later versions, a possibility was developed to compile *exe*-files and thus accelerate execution of MATLAB programs unless the latter does not include complex structures like graphic windows. It is declared for recent versions v.7.x that all such restrictions have already been eliminated. So, welcome for developing applications of your own!

Problems for Module 2.

2.1. Visualize three consequent functions given by recurrence formula

$$f_n(x) = \frac{x^{2n}}{(2n)!} \text{ for } n = 0,1,2,...$$

2.2. Visualize three consequent functions given by recurrence formula

$$f_n(x) = \frac{x^{2n-1}}{(2n-1)!} \text{ for } n = 1,2,...$$

2.3. Imagine a process governed by a law $y = e^t$. You sample however separate values of this function for certain time instances. Investigate how time increment affects accuracy of numerical differentiation by *MyDiff*.

2.4. The same question with regard to function $y = \sqrt{x}$.

2.5. Try to develop a program for numerical integration using definition of define integral from your mathematics course. How increment Δx influences accuracy of the result?

2.6.✍ Investigate problems 2.3 to 2.5 for a case that "experimental data" contain occasional errors modeled by command[14] *sigma*

[14] Look in Micromodule 3.3 or ask Help for this command.

**randn(1,1)* that generate random numbers distributed normally with dispersion *sigma=0.1*. Analyze if differentiation and integration are sensitive to such errors.

2.7. In the program *helicopter* make the stick to rotate in clockwise direction.

2.8. Develop the program that revolves "the rose" (1.9) of the problem 1.15 in clockwise or anticlockwise direction. Figure 4.6. looks similar to what you should get.

2.9. Develop a program that plots a pulsating ellipses.

2.10. Develop dialogue program that asks for *N* and revolves polygon with *N* vertices in either clockwise or anticlockwise direction. Try using *fill* command to fill in polygon with a color of choice.

2.11. Modify the program of the Example 2.2 to make the *Step01* to understand vector argument by means of loop operator *for...end*; use *plot* command. Name the new function *Step03*.

2.12. Develop program for calculating and plotting graphs of the following 'Saw-like' mathematical function

$$f(x) = \begin{cases} x+1, & if \quad x \in [-1,0] \\ 1-x, & if \quad x \in (0,1] \end{cases}.$$

2.13. Develop program that calculates values and plots graphics of the following piece-wise "trapezoidal" mathematical function that consists of 3 branches:

$$y = \begin{cases} \frac{4}{\pi}x, & if \ x \in [0,\frac{\pi}{2}] \\ 2, & if \ x \in [\frac{\pi}{2},\pi] \\ 2(2-\frac{1}{\pi}x), & if \ x \in [\pi,2\pi] \end{cases}.$$

(Hints for problems 2.12 – 2.13: students are advised to develop several programs with gradually increasing difficulty like it was for functions *step01, step02, step02* and *step_pi*, i.e. (*i*) program that understands single arguments but can *ezplot* and *fplot* for the domain given; (*ii*) program that understands vector arguments and may be used for *plot* command; and (*iii*) that for function periodically continued to the whole domain $-\infty < x < \infty$).

2.14. Develop program *MyMax* mentioned in section 2.3.2 that looks for greatest element in arrays; draw flow chart for it. Pay attention to documenting your programs by comments *%* to make them clear to other person, or even for you a year later.

2.15. Draw flow chart of the program *MyOrder* developed in section 2.3.3. Generalize the program to make it to "understand" two-dimensional input argument (matrices).

2.16. Learn "bubble" method of sorting [7-9] and realize it in a MATLAB program. Analyze, what of such algorithms already known to you is better.

2.17. Analogically to *MyDeriv*, develop dialogue program for comparing functions and their primitive (define integral) to be found "analytically", see Micromodule 1.3.2.

2.18. You certainly may solve the problem 1.20 now. Try this!

2.19. A number of exciting and useful programs may be developed by reader now. Some our students made their term papers named "MATLAB Guide to Analytical Geometry", or "Test your Knowledge of Functions by MATLAB", etc. You are advised to follow them!

Module 3: MATLAB for learning and investigation

General module characteristics: Despite you have spent rather few time for mastering MATLAB yet, you have already been able to get a significant advantage in learning your university disciplines, especially such difficult ones as mathematics and physics. It must be confessed that mathematics is often taught somewhat scholastic. However, you can now check many of its statements "experimentally". Really, many scientific facts were found by similar "numeric experiments". Few examples below may help you in doing this and encourage you for further investigations on your own.

Module structure

Micromodule 3.1. The awful "ε – δ language"!
Micromodule 3.2. Taylor, Fourier… Who else?
Micromodule 3.3. Discovering empirical formulas
Micromodule 3.4. Efficiency of programs
Micromodule 3.5. Your further discoveries with MATLAB
Problems for Module 3

Micromodule 3.1. The awful "ε – δ language"!

It is our experience in teaching that students conceive definition of the limit with difficulty; introduction of ε and δ in this definition leaves unclear to them. Well, let us try to investigate "where function or sequence tends to" by means of self-evident literal calculations by MATLAB.

1. Consider the famous "Second Famous Limit". It states that the numeric sequence $s_n = \left(1+\frac{1}{n}\right)^n$ tends to a certain border value when integer number n grows unrestrictedly. The program *limit1* presented in its listing is convenient to plot the sequence s_n step by step in a form of dialogue. Strings in the listing have been numbered for convenience in explanations.

Listing 3.1 of *limit1.m*

```
1 %This script calculates and plots numeric sequence
2 % s(n)=(1+1/n)^n known as the First Famous Limit.
3 % After each N calculations the program stops and asks
4 % if it should stop or go on in calculations. By default, N=10,
5 disp('This program investigates The Second Famous Limit');
6 disp('========================')
```

```
7    N=10;  n1=0;
8   Answer=1;
9 while Answer > 0
10       for k=1 : N
11            n(n1+k)=n1+k; s(n1+k)=(1 +1/(n1+k))^(n1+k);
12       end
13       n1=n1+N;
14       plot(n , s ,'-', n , s,'or');
15      title('Investigation if s_n=(1+1/n)^n has a limit.')
16      xlabel('Number n');  ylabel('s_n');
17      text(5, 2.1,'\bf  Use the Command Line for further commands...')
18        if n1==N
19           str0='First ';
20        else
21           str0='Next ';
22      end
23
24   disp([str0, num2str(N),' points have been added to the plot.'])
25   disp('Analyze results and deceide if I am to go on.')
26   disp ('Imput any positive number to go on,')
27    Answer=input('or a NEGATIVE NUMBER to complete!');
28   disp('------------------------');
29      if Answer < 0
30          disp('--');
31          disp('I''ve finished the work on your request. Thanks!')
32          disp('=====================================');
33      end
34  end
35  disp('Enjoy your results and draw conclusions!')
36   clear n
```

Comment to program. The program works in the following way. Being run, it explains what it serves to (*'This program investigates The Second Famous Limit'*). Having plotted fist graph for $1 \le n \le N$ for the specified $N = 10$, the program stops and asks user if it should finish or go on in calculations. Input of any positive number continues calculations and plotting for next N steps. To finish, input any negative number, say -1. This logic has been realized by the *while ... end* loop, 9 – 34. Each next N calculations are added by the *for*-loop, 10 – 12.

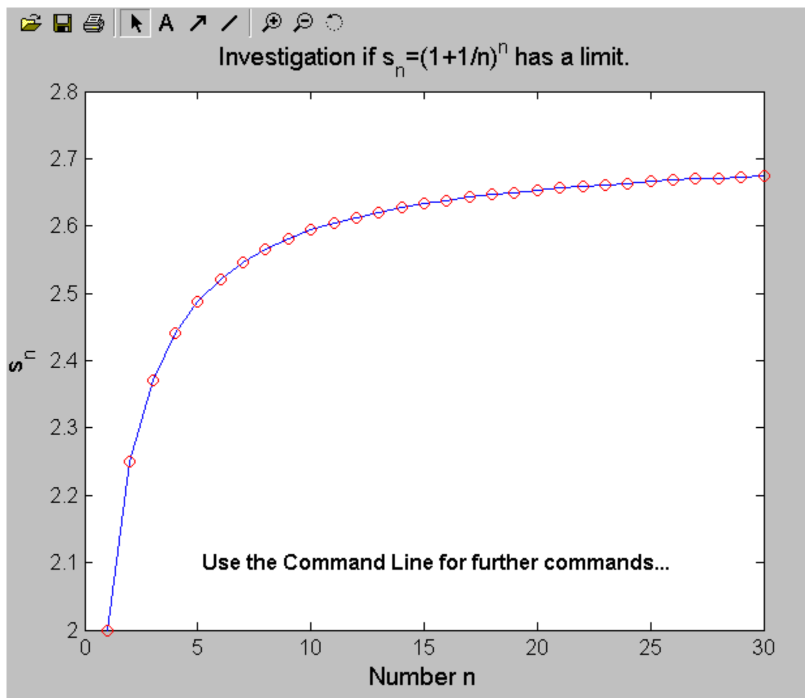


Fig. 3.1. Plotting sequence s_n of the Second Famous Limit by the program *limit1*. Analysis determines that (*i*) s_n tends to a certain limit (find it yourself!) and (*ii*) sequence elements grow monotonically.

"Logical behaviour" of text 24 is managed by the *if*-block 18 – 22. Transformation of numbers to characters by procedure *num2str*(*N*) and concatenation of strings was used here, see [1]. In concern of proper writing sub- and superscripts by the line 15, ask MATLAB for available *TeX characters* what is useful also for making text bold (command *bf* in line 17) and writing Greek characters.

Comment to mathematics. Graphical results of calculations have been shown in Figure 3.1. It is shown for the range $1 \le n \le 30$ but may be extended to any length. It may be suggested (and proved by further calculation) that s_n never exceeds certain values, say $s_n < 2.8$.

Note also that s_n grows monotonically, the fact you used in formal proof.

Numeric value that the s_n can never exceed may be defined more precisely. Do this yourself! With MATLAB, you unlikely require two hundred years as mathematicians of 16th and 17th centuries did. However, they discovered that this number is transcendental and gave a special name *e* that since has widely been used in logarithms and in mathematics at all.

Here was the first example how programming converts boring school tasks into interesting research! MATLAB's command *limit* may also be useful in learning function.

2.✍ Let us consider another problem associated with limits and sequences. The latter is called *recurrent sequence* if its nth element may be calculated by means of a previous one, of $(n\text{-}1)^{th}$, through a *recurrent formulae*. Consider quadratic formulas $s_n = s_{n-1}^2 + C$ where s_0 and C are allowed to be complex numbers. Having chosen complex initial element s_0 and complex constant C, get complex elements s_1, s_2, $\ldots, s_n$ and look where they tend while $n \to \infty$. Because they are complex, each element corresponds to a point on the plane *xOy* with coordinates $x_n = \mathrm{Re}(s_n)$ and $y_n = \mathrm{Im}(s_n)$[15].

Does such sequence have always its limit? It depends! Take different complex s_0 and C, and try. The path the sequence s_n converges or diverges[16] may be very different but often forms a strange picture. Two of them obtained for $s_0 = 0.15 + .01\mathrm{i}$ are shown in Fig. 3.2. The left one that resembles a spiral galaxy converges counter-clockwise was obtained with $C = \text{-}.05\text{-}.6\mathrm{i}$. In the right picture obtained with $C = .36 + .35\mathrm{i}$ the current point s_n jumps between five "spots" that behave as a clockwise diverging

[15] Real and imaginary parts of complex number that correspond to MATLAB functions *real*(*s*) and *imag*(*s*).

[16] These terms mean '*tends to a limit position*' and '*does not tend*' correspondingly.

galaxies on the screen. In the case $C = \text{-}0.1238 + 0.5651\mathrm{i}$

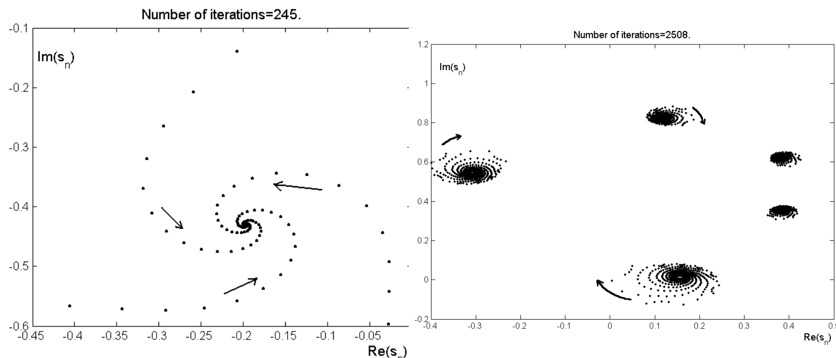


Fig. 3.2. Two plots of complex quadratic sequence that resemble one converging (left) and five diverging (right) spiral "galaxies".

"galaxies" tend to complex point $\text{-}0.225 + 3897\mathrm{i}$ along linear trajectories. You are advised to get more funny pictures! It is worth to examine constants $C = -0.8 + 0.1i$, $-0.69 + 0.12i$, $-0.7 + 0.1i$, $-0.69 + 0.12i$ and $-0.3905 - 0.5868i$ with $s_0 = 0$. Despite they are so strange, the area belongs to very recent science of fractals [7] where one could get more information.

Micromodule 3.2. Taylor, Fourier… Who else?

These are famous names that mentioned in the title. Theorems of both are studied in Higher Mathematics course. The theorems sound similar.

Taylor's theorem: for almost any[17] function $f(x)$, a polynomial $T_n(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ may be found that approximates it; the more is n the close is proximity of both.

Fourier theorem: almost any periodic function $f(x)$ may be approached by a trigonometric polynomial

[17] Precise sense of important terms "*almost any*" and "*approximate*" (or "*approach*") look in handbooks.

$$T_n(x) = a_0 + (a_1 \sin x + b_1 \cos x) + a_2(\sin 2x + b_2 \cos 2x) + \ldots$$
$$\ldots + a_n(\sin nx + b_n \cos nx);$$

proximity becomes "better" for bigger n.

Strict proof of the theorems is given in textbooks. MATLAB is useful to examine the theorems by example.

Example 3.1. Validate by means of plotting that the below Fourier polynomial

$$F_n(x) = \frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots + \frac{\sin(2n-1)x}{2n-1}\right)$$

converges to the Step-function $f(t) = \begin{cases} -1, & x \in (-\pi, 0) \\ +1, & x \in (0, \pi) \end{cases}$.

Solution of the problem gives dialogue program whose listing is given below.

Listing 3.2 of Script "*Fourier1.m*"

```
% This Command Line Dialogue Script
% plots periodical Step Function over area a < x < b provided by user
% and graphically compares it with two approaching Fourier polynomials
% whose orders n1 and n2 are also requested from user.
% Copyright Ye.Gayev, February 2006.

welcome; % to greet user
disp('=================================')
disp('This program plots periodical Step Function over area a < x < b')
disp('(you will be asked about values a and b)')
disp('and compares it with two approaching Fourier polynomials of the form')
disp('Fn(x)=4*(sin(x)+sin(3x)/3+sin(5x)/5+ ... +sin((2n-1)x)/(2n-1))/pi.')
disp('You will be asked about orders n1 and n2 of the two polynomials.')
disp('-------------------------------')
disp('Press any key to start.'); pause
% Dialogue start
disp('Please enter in brackets a and b,')
A=input('interval endpoints, to plot the functions: ');
disp('-------------------------------')
disp('Here are endpoints of the interval to plot:'); A
disp('Please provide two integers in brackets, orders of the polynomials: ')
N=input(' orders of the polynomials: ');
disp('=================================')
```

```
disp('Thanks! Now, analyze results in Figure.')

% Creating fist polynomial in symbolic form
F1=' ' ;
for i=1:N(1)
    Coefficient=num2str(2*i-1);
    Summand=['4*sin((', Coefficient, ')*x)/pi'];
    F1=[F1, '+', Summand];
end

% Creating second polynomial in symbolic form
F2=' ';
for i=1:N(2)
    Coefficient=num2str(2*i-1);
    Summand=['4*sin((', Coefficient, ')*x)/pi'];
    F2=[F2, '+', Summand];
end

ezplot(F1,[A(1), A(2)]);
hold on; fplot(F2, [A(1),  A(2)], 'g');
title('{\bf Step(x)} (red) {\bf compared with F_1(x)} (blue)
                                        {\bf and F_2(x)} (green)')
```

Comment to program. The dialogue managed by the program seems to be clear both from programming and user aspects. It begins with the command *welcome* that greets user in the way already described. Both trigonometric polynomials to plot are created in symbolic form by addition terms step-by-step. Variable *Summand* represents regular term in each loop, so that *F1* and *F2* get the form of required polynomial at the end. Command *fplot* was chosen for plotting second polynomial, as it may control (in contrast to *ezplot*) colour of lines. Using TeX commands gives the better featured title.

Comment to mathematics. Try calculations over different domains [*a, b*] and, what is more important, for polynomials of different orders and analyze results. Figure 3.3 presents graphical output for domain $[-\pi, 2\pi]$, i.e. one and half period of the Step-function, where the Step-function is compared with polynomials $F_3(x)$ and $F_7(x)$. Similar, polynomials of elder order may be compared with it. In fact, it may be concluded that trigonometric polynomials wave over both steps

of the function. The number of waves corresponds to polynomial order

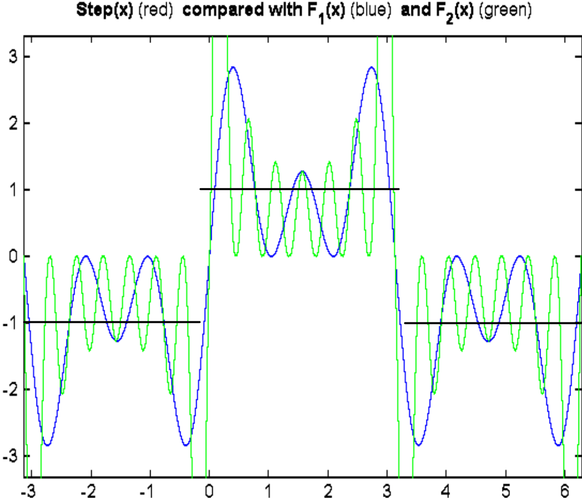


Fig. 3.3. Graphical output of program *Furier1* allows investigation Fourier polynomials that approximate Step function. Result for $n_1 = 3$ and $n_2 = 7$ (see Problem 3.4, then).

n. It may be thought that none of them "approximate" the function, even if $n \to \infty$. Higher mathematics introduces however another idea of "proximity of two functions" in terms of integrals (areas). And this proximity between the Step-function and $F_n(x)$ tends to zero while $n \to \infty$, what turned to be very useful in mathematics, signal processing and informatics at all.

Investigation of the Taylor' theorem is advised to carry out yourselves. MATLAB's command *taylor* is useful in finding polynomial expansions "analytically".

Micromodule 3.3. Discovering empirical formulas

This is a common problem in physics, sociology, economical science etc. to look for analytical formulas for representing experimental (or obtained by other means) data. MATLAB suggests a wonderful tool for solving this problem called "*curve fitting*".

Let an observation was carried out in time moments *t=1:.5:6*. Imagine, we know (but nobody else!) the law $y = at^{\alpha} + b$ with coefficients, say $a = 2$, $\alpha = 0{,}3$ and $b = 1$, the process $y(t)$ is governed by. If one would sample values of y in given moments, results were certainly different from those theoretical values because of measurement errors. How to get "experimental" data to learn an

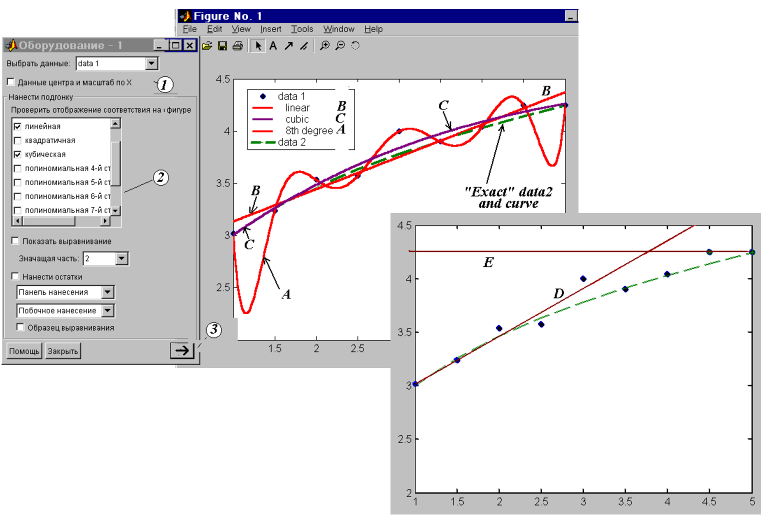


Fig. 3.4. Figure Window (middle) with exact "experimental" data and their linear (B), cubic (C) fits and that of 8th order (A) found via *Fitting Interface* (left). Right: two possible straight lines going exactly through two "experimental" points.

example? MATLAB allows a "numerical experiment". The command *rand*(*1,n*) can generate vector *1*n* of random numbers uniformly distributed between 0 and 1. However, use the command *Eps*randn*(*1,n*) that generates *normally distributed random numbers* with dispersion *Eps* as it always happens in any measurement. So, the following commands produce theoretically "exact values of *y*" Y_{exact} along with their "experimental" representatives Y_{exp} with "stochastic errors":

```
>> t=1:.5:5; a=2; Alpha=.3; b=1; Yexact=a*t.^Alpha+b;
>> Eps=.1; L=length(t); Yexp=Yexact+Eps*randn(1,L);
```

>> *plot*(*t,Yexp,'o', t,Yexact,'--'*)

The last command results evidently in plotting dashed "theoretical curve" and "experimental" circles[18], Fig. 3.4; the more is *Eps*, the more the circles are scattered around the curve. The number of "experimental circles" *L* has been determined by *length*(*t*). The *curve fitting problem* sounds: find equation to represent experimental data. Once we have it, we could *interpolate* experimental data (i.e. get $y(t)$ between measured points), or even *extrapolate* them (i.e. estimate $y(t)$ in the past, or forecast $y(t)$ for future). Note that problem formulation is not addressed to finding precise law of the process what should be looked for in particular scientific discipline; the problem lies in selection among specified set of functions such as linear $y = p_1 t + p_2$, polynomial $P_n(t) = p_1 t^n + p_2 t^{n-1} + ... + p_n t \ + p_{n+1}$, exponential or other functions [3,4].

To start our analysis, choose ***Tools*** ‣ ***Basic Fitting*** in the Figure menu. The *Basic Fitting Interface* that appears is shown in the Fig. 3.4, left on the Figure window. First, choose *data1* in the window *1* as the *data2* relate to "exact" values of the function and will be used for comparison only. Secondly, we should "experiment" with type of the approximation: functions from linear to 10th grade have been suggested.

Try polynomial of 8th grade; you'll receive a waving curve labelled *A* in the Figure. Despite this curve goes exactly through all *L*=9 given points, it may unlikely be accepted as being good. It is clear why it goes exactly through the points: because polynomial of the above grade contains 9 indefinite coefficients, and thus there are exactly 9 equations for determining them. That is why we receive warning window "*Polynomial is not unique: degree >= number of data points*" if we would choose 9th or 10th degree. Coefficients of the polynomial found might be read in additional window that appears if we would press arrow *3*. You understand that there are an infinite number of solutions in the latter cases![19]

---

[18] "Experimental" points may lay differently in your figure as they include "random" additions! The same is valid for straight lines in the right figure.

[19] More valuable information may be found in MATLAB HELP ‣ ***Mathematics*** ‣ ***Case Study: Curve Fitting, Polynomial Fit, Analyzing Residuals***.

Learn polynomials of fewer degree now. Having chosen linear polynomial, we receive the line *B*. Straight lines are often chosen for representing experimental data especially in cases when nothing is known about real behaviour of the process or accuracy of data is very poor. We could decide in our case however that polynomials of second or third degree better represent our "theoretical law" as they are convex like our process is. Such practical ideas often influence final decision of the problem.

These were not all questions that may posed by a critical mind. In fact, how were determined polynomials *B* and *C* if the number of equations, 9, is more than indefinite coefficients in the polynomial form? Underlying mathematical principles are so important for mathematics and informatics that we devote a few space to them.

Go on with our criticism. One could choose any two points (among nine!) to ensure uniqueness of solution while getting straight lines. However, choosing first and second points, or the last point and next to last

```
>> t1=[t(1), t(2)]; Y1=[Yexp(1), Yexp(2)];
>> t2=[t(L-1), t(L)]; Y2=[Yexp(L-1), Yexp(L)];
>> figure; plot(t1,Y1,'*', t2,Y2,'p')
```

results in completely different straight lines *E* and *D*, right picture in the Fig. 3.4. Which is the best? Note also the "*Norm of residuals*" in the window for coefficients – what does it mean?

A very fruitful mathematical approach lies in the following *Least Squares Method* that finds straight line "equally good" for all the points it represents. Take any straight line $y = at + b$ with coefficients a and b undefined at this stage. This function being calculated at any knot t_i produces function value $y_i = at_i + b$ that, naturally, differs from correspondent "experimental result" at this point $Y_{i,exp}$ so that the residual is positive or negative or, seldom, zero. *Norm of residuals*, i.e.

$$N(a,b) = \sum_{i=1}^{N} \left(Y_{i,\exp} - at_i - b\right)^2$$

may be a good measure of how all the experimental points depart from the line. The norm is always nonnegative and depends on both coefficients a and b so that we can choose them to minimize the norm.

$$\frac{\partial N}{\partial a} = -2\sum_{i=1}^{N}\left(Y_{i,\exp} - at_i - b\right)t_i = 0,$$
$$\frac{\partial N}{\partial b} = -2\sum_{i=1}^{N}\left(Y_{i,\exp} - at_i - b\right) = 0.$$

The latter equalities lied to two linear equations with respect to coefficients a and b:

$$a\sum_{i=1}^{N} t_i^2 + b\sum_{i=1}^{N} t_i = \sum_{i=1}^{N} Y_{i,\exp}\, t_i ,$$
$$a\sum_{i=1}^{N} t_i + bN = \sum_{i=1}^{N} Y_{i,\exp}\, t_i .$$

Note that we receive as many equations as there are coefficients irrespective on how many experimental points we have. It is strictly proved that the equation set has always unique solution, [4]. The latter is what is found by Curve Fitting graphical program. You could find it yourself by one of MATLAB's commands for systems of linear algebraic equations (SLAE), Examples 1.11, 1.12.

Similar, one gets SLAE with *K* equations for fitting data by polynomial of (*K-1*)th degree, [2,4]. Many useful functions to fit data such as exponential and logarithmic should easily be previously transformed to get linear equations for their coefficients. There are also functions that lead to non-linear problems. For them, MATLAB contains special program *cftool*.

You get familiar with curve fitting program and learned its mathematical background, the Least Square Method. You learned also programs *rand* and *randn* for generating random numbers. It is to emphasize that the fitting program is the first program in this book with a *graphical interface* (*GUI*). In the Module 4 you shall learn more how to create your own GUIs.

Micromodule 3.4. Efficiency of programs

Once created, your program becomes an object of exchange with other programmers, to sell and, what is especially important for the moment, to investigate. Some programs are "better" or "worth" than others. In what sense are they? Among many criteria, *efficiency of program*, i.e. how fast it works, is one of most significant.

Assume the program of interest works with square matrices of order n. Let us investigate how much processor time it consumes with respect to n. MATLAB's commands *tic* and *toc* start stopwatch and print elapsed time (in seconds) correspondingly, and thus may be used in our problem. Let a script *generator* generates matrices *A*, *B*, *C*, … of variable order *n*, and script *to_test* contains codes whose effectiveness is to be examined. The following command from the Command Line

```
>> n=2, generator; tic; to_test; T(n-1)=toc;
```

produces matrices 2x2, performs them and assigns the time spent to the array element *T*(*1*). Next command

```
>> n=n+1, generator; tic; to_test; T(n-1)=toc;                    (3.1)
```

does the same for arrays of 3rd order and saves the elapsed time in *T*(*2*). (The number *n* is printed on screen to observe the process). Repetition of (3.1) from the Command Line gets *T*(*3*) for matrices of fourth order, and so on. Finally, dependence *T* versus *n* may be plotted,

```
>> plot(T, 'g')                                                   (3.2)
```

Now, let us take the program *My1Order*, Listing 2.9A, to test as an example. Few comments before analyzing it. Because modern computers have already been extremely fast, it is worth to produce ten matrices rather than a single in *generator.m*. To generate them automatically, several structures were programmed. Matrices *A*, *B* and *C* were of structure:

$$A = \begin{pmatrix} 1\ 2\ 3 \\ 2\ 3\ 4 \\ 3\ 4\ 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 1^2 & 2^2 & 3^2 \\ 1^3 & 2^3 & 3^3 \end{pmatrix}, \quad C = \begin{pmatrix} 1\ 2\ 3 \\ 4\ 5\ 6 \\ 7\ 8\ 9 \end{pmatrix},$$

for which one easily can suggest the rule for any arbitrary order *n*, and matrix *D* was generated to consist of random integer numbers uniformly distributed between 0 and 10 as *D*=*round*(*10***rand*(*n*)). Six more matrices of the same order *n* were taken as *E*=*B***A*, *F*=*C***A*, *G*=*D***A*, *H*=*C***B*, *I*=*D***B* and *J*=(*D*+*B*)**A*. Here is the code:

Listing of *generator.m*

```
% Generator
% is a Script to produce matrices n*n for a given integer n:
% A=[1 2 3 ...; 2 3 4 ...; 3 4 5 ...; etc.   ]
```

```
% B=[1 2 3 ...; 1^2  2^2  3^3 ...; 1^3  2^3  3^3 ...; etc.]
% C=[1 2 3; 4 5 6; 7 8 9] and so forth for n>3
% D=10*rand(n)
% to test Time Execution of programs working with matrices

clear  A  B  C  D  E  F  G  H  I  J
for i=1:n
     A(i, :)=i : n+i-1;  B(i, :)=(1: n).^i ;
     C(i, :)=n.*(i-1)+1:n*(i-1)+n;
end
D=round(10*rand(n));
E=B*A;  F=C*A;  G=D*A;  H=C*B;  I=D*B;  J=(D+B)*A;
% End of generator.m
```

So, the program *to_test.m* includes application of the same program *My1Order* to perform all the ten matrices:

Listing of *to_test.m*

```
% Script to_Test
% to examine for Execution Time programs like My1Order.m
% that work with matrices (produced by Generator.m)

% Analysis of my program
A1=My1Order(A,'Decrease');   B1=My1Order(B,'Decrease');
C1=My1Order(C,'Decrease');   D1=My1Order(D,'Decrease');
E1=My1Order(E,'increase');    F1=My1Order(F,'Increase');
G1=My1Order(G,'increase');    H1=My1Order(H,'Increase');
I1=My1Order(I,'increase');    J1=My1Order(J,'decrease');
% End of  to_test.m
```

Commands (3.1) may easily be issued by hands 100 or even 200 times so that statistics for execution times will be collected in the array T. Plot the latter as (3.2) and note that function $T(n)$ grows with n. Using *Basic Fitting Window* (see previous Micromodule 3.3 and Fig. 3.4), interpolate the data T as a second order polynomial[20]; it is for computer of authors $y = 0.00052x^2 - 0{,}0027x + 0{,}05$.

---

[20] Your own program might be employed as well, see problem 3.9.

Next, let us test intrinsic MATLAB's function *sort* that works similar to *My1Order* but re-orders columns elements rather than that of rows; to do this, replace words *My1Order* by *sort* in the program *to_test.m*. Here is an example of replacing:

*A1=sort(A')'; B1=sort(B')'; J1=sort(((D+B)*A)')';*

(transposed matrices are used in the case of *sort*). Save results in an array *T1* by commands (3.1). Finally, commands

```
>> plot(1:200, T(1:200), 'g', 1:200, T(1:100), 'r')
>>title('Efficiency of Sorting Algorithms')
>> legend('Efficiency of My1Order.m ', 'Efficiency of sort.m')
>> xlabel('Matrix order, n'); ylabel('Time ellapsed, sec');
```

let us compare both programs. The program *sort.m* turns to be much faster as seen from Fig. 3.5. It begins to rise at only $n \approx 140$ but its growth is steeper. In fact, applying *Basic Fitting* gives equation of its rising as $y = 0.0016x^2 - ...$ (Fig. 3.5).

These results mean that the program *sort.m* is significantly more effective in the domain of *n* investigated. No wonder: better mathematicians were employed in developing MATLAB! However, because its growth is about $\frac{0,0016}{0.00052} \approx 3$ times more intensive than another one, situation may change for especially large matrices.

It is clear that leading coefficient plays a prime role in estimating function growth, and thus is commonly used in characterizing efficiency of algorithms. It is quite well, if its number of operations (so, execution time *T*) rises as n^1. It is much worth if $T \propto n^2$ or $T \propto n^3$. Programs have been almost unacceptable if $T \propto e^n$. Computer science experienced even a revolution in late 1960th when algorithms of Fourier Transform with execution time proportional to n^2 were changed to algorithms of Fast Fourier Transform with $T \propto n \ln n$. So the IT science needs to pay sufficient attention to the properties of programs discussed. Correspondingly, MATLAB includes a special tool for this, *Profiler*.

Micromodule 3.5. Your further discoveries with MATLAB

It is our hope that you enjoyed discoveries or re-discoveries you've done with MATLAB in this book. But it is the only start to you! What about

playing games of chance with MATLAB or, better, researching probability problems associated with them? Look into our paper [22]! Or, would you like to play or even write music with MATLAB? If so, read our paper [21]. Note that all those beautiful programs were made by first year students. They are on your forces, too! With MATLAB.

Problems for Module 3

3.1. "Visualize" and investigate the "*First Famous Limit*" $\lim_{x\to 0}\frac{\sin(x)}{x}$.

Hints: (*a*) consider separately the "left limit" $x \to 0+0$ and the "right limit" $x \to 0-0$; (*b*) as x is continuous argument, consider different "tendencies" of x to 0 ± 0, for example $x = \pm 1/n$, or $x = \pm 1/n^2$ with $n \to \infty$. Is there any difference?

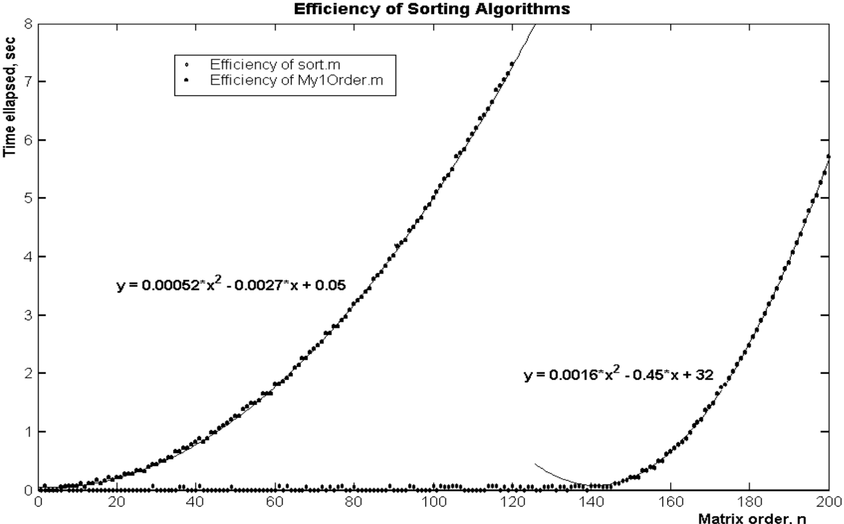


Fig. 3.5. Comparison of execution time for two array sorting algorithms: dots are "experimental" results; lines are fitting curves.

3.2. Make "experiments" with numeric series $S_n = \sum_{1}^{n} (-1)^i \left(\frac{2i+1}{3i+1} \right)^i$ to discover if it converges to any limit with $n \to \infty$, [5].

3.3. Determine "experimentally" whether $S_n = 1 - \frac{2}{7} + ... + (-1)^n \frac{n}{6n-5}$ converges to any limit while $n \to \infty$, [5].

3.4. Plot of Step-function in the Fig. 3.3 was made manually. Modify the program *Fourier1* to compare consequent functions *F1* and *F2* with also Step-function (Micromodule 2.3.1). Try using *plot* instead of *ezplot* and *fplot*.

3.5. Analyze by means of a dialogue program if the following Fourier series

$$f(x) = \left(\frac{\pi}{8} - \frac{1}{2} \right) - \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{\cos(2k+1)x}{(2k+1)^2} - \frac{1}{4} \sum_{k=0}^{\infty} \frac{\sin 2(k+1)x}{k+1} +$$

$$+\sum_{k=0}^{\infty}\left(\frac{2}{\pi}+\frac{1}{2(2k+1)}\right)\sin(2k+1)x$$

converges to discontinuous function

$$f(x)=\begin{cases}-1, & -\pi<x<0;\\ \dfrac{x}{2}, & 0<x<\pi.\end{cases}$$

3.6. By means of *taylor* find Taylor's expansion of function $\sin(t)$, write down its *pretty*-representation and investigate if it really approaches the given function for several n.

3.7. Plot the Nth partial sum $S_n(x)=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}$ and compare it with the function $y=e^x$ over domain $a\le x\le b$. Develop a dialogue program for such investigation of the Tailor's theorem.

3.8. Generate "experimental" data randomly dispersed over linear function $y=-.7x+3.2$. Create your own dialogue program for curve fitting to these data by Least Squares Method.

3.9. Create your own dialogue program for fitting curves of second order. Compare results with the MATLAB program.

3.10. Investigate how much time is required for calculating elementary functions like *sin*, *tan*, *atan* etc. in MATLAB. (Hint: for various *n* produce table of an argument *N=2^n;x=0:.1:N* and calculate functions for it; use *Basic Fitting* Tool to estimate functional dependence between the time and the number of arguments *10*N+1*; linear grows is expected).

3.11. Investigate how much time is required for calculating *error function erf*(*x*) in MATLAB. (Hint: because exponential grows is expected, it may be worth use of logarithmic coordinates {*lg N*, *lg Time*} to determine the fitting curve).

Module 4: Graphical User Interface in MATLAB

General module characteristics: As you already can develop your own programs and apply them to disciplines you study in university, you certainly would be happy if your programs looked as nice and attractive as the Windows, the Word and other programs of high standard level. It means that the Graphical User Interface, the GUI, is what you just need.

Module structure

Micromodule 4.1. Graphical User Interface (GUI) standards

Micromodule 4.2. Games with MATLAB GUI elements

4.2.1. *menu* command

4.2.2. *uicontrol* commands

Micromodule 4.3. *guide*, MATLAB GUI developer

Micromodule 4.4. An example: GUI for *helicopter*

Conclusion

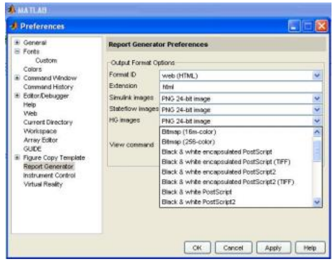
Problems for Module 4

Micromodule 4.1. Graphical User Interface (GUI) standards

Earlier, in the Module 2, we created quite intellectual programs with a dialogue between them and the user from the command line. In the future, however, computer programs will communicate with us by a human voice and supply results in a visual or audio form convenient to us. At the moment, it is widely used to create GUIs (to pronounce as ['gu:i]) for providing users with aesthetically pleasing communication interface. So, development of them becomes an important part of modern engineering education now, [11,13,16]

The most famous examples of GUIs are Microsoft Windows and Word. They employ various *menus*, *text windows* and *graphical windows*, *buttons, radio buttons*, *check boxes*, *taskbars*, *pop-up menus*, *sliders* to be affected either by keyboard of by mouse. Many popular articles about history and types of GUIs may be found in Internet [16,17]. MATLAB itself provides you a friendly graphical interface; its elements with explanations have been shown in the Fig. 4.1.

Another good example of GUI is MATLAB's Basic Fitting Interface, see Micromodule 3.3.



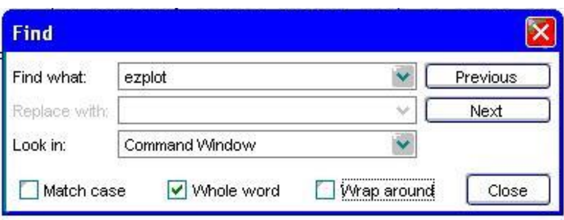


Fig. 4.1. Examples of MATLAB 7.1 GUIs: 1 – *menus*, 2 – *text windows*, 3 – *buttons*, 4 – *radio buttons*, 5 – *check boxes*, 6 – *taskbars*

Micromodule 4.2. Games with MATLAB GUI elements

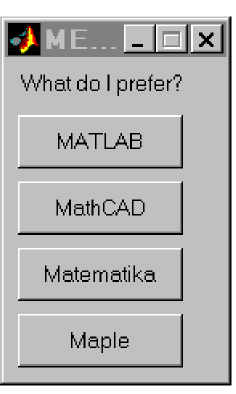
Before starting to learn GUI, it is useful to learn some GUI elements in a form of a play with them. Students are advised to follow the plan below and to make experiments of their own.

4.2.1. *menu* command

First, try the command *menu*

>> *MyChoice=menu('What do I prefer?','MATLAB', 'MathCAD', ... 'Matematika','Maple')*

and look what happens. You receive a nice menu of gray color like in Fig. 4.2, left. Note that the *Command Line* remains busy. Any of four *pushbuttons* in the menu just created may be pressed; the variable *MyChoice* gets a value 1 to 4 depending on the button chosen, and the menu *What do I prefer?* disappears. You need to repeat the command above to go on our experiment.



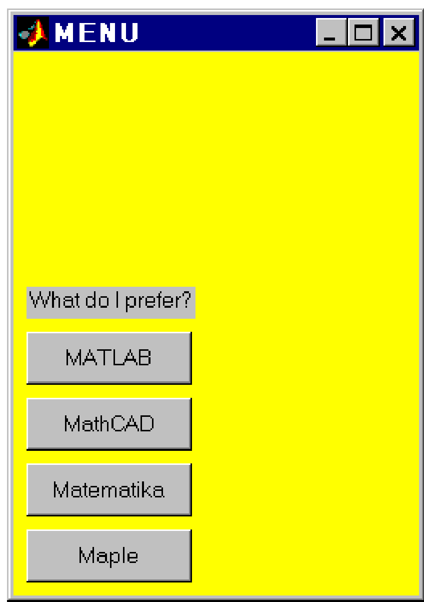


Fig. 4.2. Two graphical objects created by *menu* command. Colors and dimensions may be changed by *set*.

After <*Ctrl* - *C*>, graphical object makes the *Command Line* free[21] (don't mind error messages) but does not disappear; even several new such menus may be run in such a way. Try: it is supposed you cannot change size of them at the moment.

Now, play with the command *gcf* (*G*et *C*urrent *F*igure): different integers will be obtained with regard to the menu to be chosen current. These integer numbers are *handles* to particular existed objects. By the command like

>> *set*(*2*, *'color'* , *'yellow'*)

where *2* is handle of a particular Figure, you can set a color you wish to the menu you wish[22].

---

[21] This is true for the version 6.* while MATLAB v.7.* behaves in a different way.

[22] Function *set* was already used in listing 2.3.

How could we address a particular object? What properties are we able to change, and how to do this? As we know handles to each object, the command
>> *get*(*IntegerNumber*)
brings to light a long list of properties and their current values relevant to the object with the *handle=IntegerNumber*. You are advised to try next games with them by the command *set*:

1. Change color of any menu:
>> *set*(*3,'color',yellow'*)

2. Change the name of one of your menus, for example
>> *set*(*2, 'Name', 'My discovery'*)
You cannot see the new title *My discovery* in full, don't you? Go on!

3. Re-switch the property *Resize* from *off* to *on*:
>> *set*(*2, 'Resize', 'on'*)
You can change size of your rectangle menu by means of mouse now!

4. And a hocus-pocus at the end: by the commands
>> *set*(*3*, '*Visible* ', ' *off* ')
>> *set*(*3*, '*Visible* ', ' *on* ')
your graphical object number 3 will disappear and appear again!

That's all for the *menu*. Feel free to make experiments of your own with graphical objects! However, how to know values to assign to a particular property, for instance for *Pointer*? The command *set* with no value shall prompt you:

>> *set*(*1*, ' *Pointer* ')

With only handle, the command displays the whole list of values,

>> *set*(*1*)

There is a broad freedom in controlling your graphical properties. Feel free to make experiments with also properties *TooltipString*, *Pointer*, etc. Only *read-only properties* cannot be accessed.

4.2.2. *uicontrol* commands

Many names of commands in our focus start with *ui* what abbreviates the words *User Interface*. Like before, it is also useful to play with the command *uicontrol* and investigate its capabilities in such way.

1. Big number of MATLAB' experiments is possible with the syntax

figure(*FigNo*); *FigNoControlNo* = *uicontrol*(' *Style* ', 'ControlStyle ')

that creates an "empty" Figure with the number *FigNo* and with a *Control* that may be one of the *styles* listed and briefly explained in the table:

Style of a control	Brief explanation
checkbox	A rectangle for making choice *yes* or *not* by mouse
edit	Provides a field for editable text
frame	A rectangle to visually group some items
listbox	Control to display a list of items defined by uses
popupmenu	To display a list of choices user wishes
pushbutton	Button that simulates pressing and depressing
radiobutton	Similar to *checkbox*
slider	Provides a sliding bar
text	Rectangle with an unchangeable (static) text
toggle	Control to execute *callback* when clicked

For example, create Figure 5 with pushbutton that gets its handle *Fig5Obj1* (it doesn't matter what of the numeric value it gets):

```
>> figure(5 ),    Fig5Obj1 = uicontrol( ' Style ' , ' pushbutton ')
```

It doesn't matter what of the numeric value the handle gets; it is more important to be able to get properties of this first object in the Figure 5:

```
>> get(Fig5Obj1)
```

Note that controls of different style have different properties! Particularly, find the way to manage dimensions and position of your controls.

2. Learn what properties the above controls have, and try to manage them. For example, the following command sets green color to the above pushbutton:

```
>> set(Fig5Obj1 ,' BackgroundColor ', ' green ')
```

The next command superscribes the word *Start* by font size defined by second pair of arguments:

```
>> set(Fig5Obj1 , ' String ' , ' Start ',' Fontsize ', 10)
```

No matter how many properties are set by a single command *set* at once.

3. Having created several Figures, try to distinguish their handles from that of controls they have. Handles of Figures allow controlling their properties. Say, the default colour of the Figure 5 may be changed:

```
>> set(5, ' color ', 'white ')
```

Try also to change properties of particular control in particular Figure.

4. Students are advised to investigate themselves other MATLAB commands for creating user interfaces, such as:

uigetfile, *uigetdir*, *helpdlg*, *uisetcolor*, *uisetfont*,
h=waitbar(*.3*,'*title*','*CloseRequestFcn*','*delete*(*h*)'),
questdlg, *msgbox*, *warndlg, errordlg*.

Micromodule 4.3. *guide*, MATLAB GUI developer

The commands you learned over may help you in creating your own graphical user interfaces to applied programs of your own. However, you are advised to use the command *guide* that implements them to facilitate such your work. MATLAB will thus make your work easier and much less time consuming than if you would work with C^{++} or Java.

Calling this program from the Command Line results first in appearance of a pilot window *Quick Start* that allows you to choose from several pre-formatted GUI templates (with UIcontrols, with Axes and Menu, and with a ready Modal Question Dialogue), Fig. 4.3. However, start with a Blank GUI template now.

A *GUI Layout Editor* appears. It consists of resizable *GUI Layout Area* (see Fig. 4.4) with a mesh for aligning objects, horizontal set of instruments at the top (labelled as A, …, D) and vertical component palette on the left with all possible GUI components (numbered here from 1 to 11 and explained in caption below). Note that pointing mouse cursor to any instrument icon or *GUI Component* (without pressing them) results in displaying a hint what the icon serves to.

Press any component 1 to 11 in the *Component Palette* and drag it in the *Layout Area*. In the Fig. 4.4 all kinds of UI controls have been placed in the Layout Area, one of each kind, to demonstrate how they look like. The grid is to align controls with respect to each other. If you find that the Layout Area is too small, resize it by mouse cursor and the *Figure Resize Tab*.

Double click any control in the *Layout Area* (or activate it and press the *Property Editor* icon labelled as C in the Fig. 4.4) to call out *Property Inspector*. Its particular values like self evident *Background Color*, *Font Name*, *Font Size* allow to simply change properties of the correspondent UI control. Esthetical feeling is required to make this part of the work.

All the controls in the Layout Area are "dead" at this stage. Press *Run Button* D, , to activate the GUI. You will be asked first about the name to save it; the name *TestGUI* is suggested. A real GUI, an object *TestGUI.fig* (with extencion *.fig*) has been created[23] with already "alive" GUI controls that simulate pushing or toggling buttons 1 and 2, enabling or disabling check box 3 and radio button 4, inputting text or numerals in the *Edit Window* 5, sliding in 7 and unfolding the list box 9 and pop-up menu 10, see Fig. 4.5. However, they do not do any useful work but rather simulate it. Axes 11 do not display any useful information as well. To do any useful actions, *callbacks* are to be associated with the UI controls. They are to be written in *m*-file.

The *m*-file *TestGUI.m* appears when the *Run Button* D is pressed for the first time; alternatively, it may be called by pressing icon B, , of the *m*-file Editor. It may be noted that this program has been quite long, of about 200 lines. Let us analyze its structure.

The program begins with declaration of our function *TestGUI* that links an output parameter *varargout* with input parameter *varargin* (abbreviations from "variables of out-argument" and "variables of in-argument"). It may be a good idea to structure this complex code in the following way.

(I) First, there is a portion of comments with some explanations. Look how they have been fitted to the names you introduced! Programmer is advised to modify comments by providing required information what his/her program serves to.

(II) Then, there are 18 lines with an initialization code that are forbidden to edit.

[23] Another way to call out the GUI already created is execution *testgui* from the Command Line.

(III) There are also two functions *TestGUI_OpeningFcn* and *TestGUI_OutputFcn* that you are advised not to modify as well.

(IV) The last and most important portion of the code includes description of all the dynamic GUI controls[24] that were designed by programmer. Some of the latter require one function to operate with (they are *Push Buttons*, *Toggle Buttons*, *Radio Buttons* and *Check Boxes*), another require two functions (*Edit Text*, *Sliders*, *List Boxes* and *Pop-Up Menus*). This is just the code portion where programmer should provide his/her own commands the program is to execute.

All this is illustrated by an example below.

Micromodule 4.4. An example: GUI for *helicopter*

Before developing GUI for any application, programmer is to plan what UI controls the program should have to communicate with user. Let us illustrate this by creating GUI to the program *helicopter*, Example 2.1.

Recall that the program *helicopter* rotates a stick on PC screen, and we may control (*i*) number of rotations *N*, (*ii*) direction of rotation (either clockwise or counter-clockwise), (*iii*) speed of the rotation (from slow to fast), and (*iv*) colour of the rotated stick. One may think the following *UI styles* to fit best for providing these data to the program: *Edit Window* for (*i*); *radiobutton* for (*ii*); *slider* for (*iii*), and *pop-up menu* for (*iv*). Resulting *GUIhelicopter* program shown in the Figure 4.6 has been made through the following stages.

1. UI control of the *Static Text* style was located on the Layout Area to make title of the future program. After doing this, double click on the newly created control, or press the Property Editor icon C, and (*a*) choose *Background Color* you like; (*b*) input title you'd like to see instead of predefined *String = Static Text*, for example "***My first GUI: Rotation of a stick. Input information required to Start***". It is important to make this title clearly understandable for users prospected! The programmer is able also to specify (*c*) *FontSize* and *FontName* (*14* for *MS Sans Serif* were chosen as well as *ForegroundColor= white*).

---

[24] i.e. *Static Text* controls are not present here.

2. Make *Axes Window* and align it with the previous control. Call the Property Editor and note how many properties has this control! *XTickLabel* and *YTickLabel* properties may be set from default values to empty in our case. Many of them may be set later.

3. Create *PushButton* and, by calling the Property Editor for it, specify its color, *String* ("Start" was used) and *FontSize* were specified such as at the stage 1. In contrast to previous steps, providing name for the *Tag* (or noting its default name like *pushbuttonN*) is required to find it in corresponding *m*-file later. In our case, the name *PushButtonStart* was associated with *Tag*.

4. It is useful to provide titles for controls introduced on below stages to explain them. The following *Static Text* controls were used: "*Number of rotations*", "*Direction of rotation*", "*Choose color of the Stick*" and "*Speed of rotation*".

5. For inputting *N*, the number of rotations, locate *Edit Text* Window on the Layout Area below Axes. As before, specify properties controlled by Property Editor. Especially important is to assign a proper name to *Tag*; we set the *Tag* to *EditN*.

6. Place radio button nearby, specify text on its panel and, especially important, set its *Tag* to, say, *RadioButtonDirection*.

7. Copy the last pair of controls (i.e. highlight it, <*Ctrl-C*> and <*Ctrl-V*>), align the new pair on the Layout Area but change their properties: title the *Static Text String* as "*Choose color of the Stick*", change the *Style* property of the radio button control to *Style=popupmenu*. The *Tag* was named as *PopUpColor*. It is important specifically for pop-up menus: in the *String* box several lines were written to list possible colors of the stick: *Red*, *Green*, *Yellow*, *Blue*, *Black*, *Magenta*.

8. Finally, place *Slider* and Static Text "*Speed of rotation*" below the slider, name its *Tag* as *SliderSpeed*. Important specifically for sliders: *Min* and *Max* properties may be set from their default values (0.0 and 1.0) to another ones. Design of the GUI *GUIhelicopter* has been completed. Save all the changes under this name by pressing *Run Button* D. Graphical structure of all the components implemented here may be overviewed by pressing *Object Browser* D, .

Now, links between all the components chosen for the new program are to be provided, and callbacks specified. The *m*-file Editor

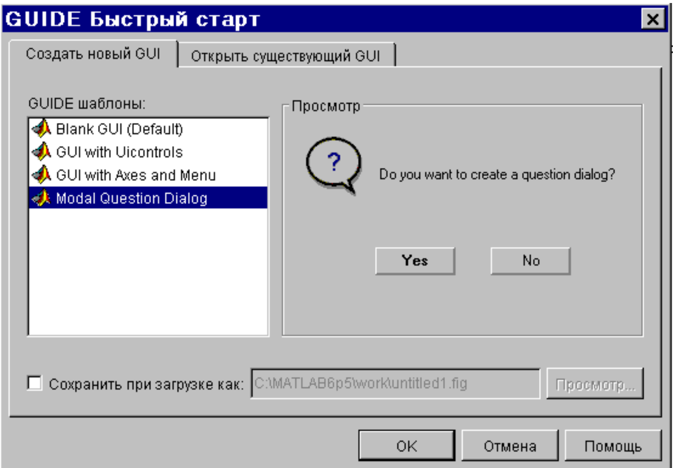


Fig. 4.3. Pilot window after calling *guide*.

with the *m*-file *GUIhelicopter.m* appeared when the *Run Button* E was pressed for the first time. It may be also called by pressing icon B, . The IV part of the generated program *GUIhelicopter.m* see in the Attachment A2. Go on with the new program!

9. In the IV part of the *GUIhelicopter.m* code find the function responsible for the push button. Accounting for the *Tag* we provided for it on the stage 3, it has been the function *PushButtonStart_Callback*. At the moment, this function contains no commands except comments. Let us write down and store the following piece of code taken from the program *helicopter* but modified:

```
N=5;  Direction=1;   ColorText='red ';
% global  N  Direction  Speed
Dt= Direction*pi/20;
for t=0 : Dt : Direction*2*N*pi
    x1=cos(t); y1=sin(t);
    x2= - x1; y2= - y1;
    Pl=plot([x1, x2], [y1, y2]);
   set(Pl, 'linewidth ',4); set(Pl, 'Color ',ColorText)
     set(Pl, 'linewidth ',4)
    axis([-1.1    1.1    -1.1    1.1]) %Constant Scales!
    set(gca,'XTick', [ ]); set(gca,'yTick ', [ ])
```

```
    hold on; plot([0],[0],'o'); hold off; % Symbol and Color of Center Point
    pause(.1)   % Speed of Rotations
%    pause(.1/Speed)  % Speed of Rotations modified
end
```

You may examine that our GUI program starts working: pressing button ***Start*** results in appearance of a stick that rotates. At this stage, however, rotation of the stick and its color do not depend on data in other UI controls. For example, symbol *N* in the window "*Number of rotations*" may be substituted for, say, 100, but stick will anyway rotate *N=5* times as this has been prescribed by the first command.

10. Find now two functions responsible for the above editable text. They are the functions *EditN_CreateFcn* and *EditN_Callback* in the code as this determines by the *Tag* set on the stage 5. The second function seems to be responsible for the callback. Two ready hints in its comments suggest to place following commands here

```
global N
Ntext=get(hObject,'String');
N=str2double( get( hObject,'String' ));
```

The second command accesses the property *String* of the current object and assigns to it symbols introduced in the window. The next command converts these symbols (they are so to be numerals!) into the number. The first command declares the introduced *N* to be global. Finally, declaration *global N* should be also introduced in the code of previous stage 9: simply comment its first row and uncomment the second. An exchange of *N* between two sub-programs has been managed in such a way. Now, the stick will make as much revolutions as user may wish.

11. A hint in the function *RadioButtonDirection_Callback* associated with our radio button accordingly stage 6 suggests the following commands to place there:

```
global Direction
Direction=(-1)^get(hObject,'Value');
```

The second command produces a number *Direction* equal either +1 or -1 depending on the radio button is labeled or not. This number is global both for the current function and for the code written on stage 9.

12. Similar to the stage 10, there are also two functions *PopUpColor_CreateFcn* and *PopUpColor_Callback* responsible for choosing color of the stick accordingly to the Tag set on the stage 7. The last one is suggested to be extended by commands

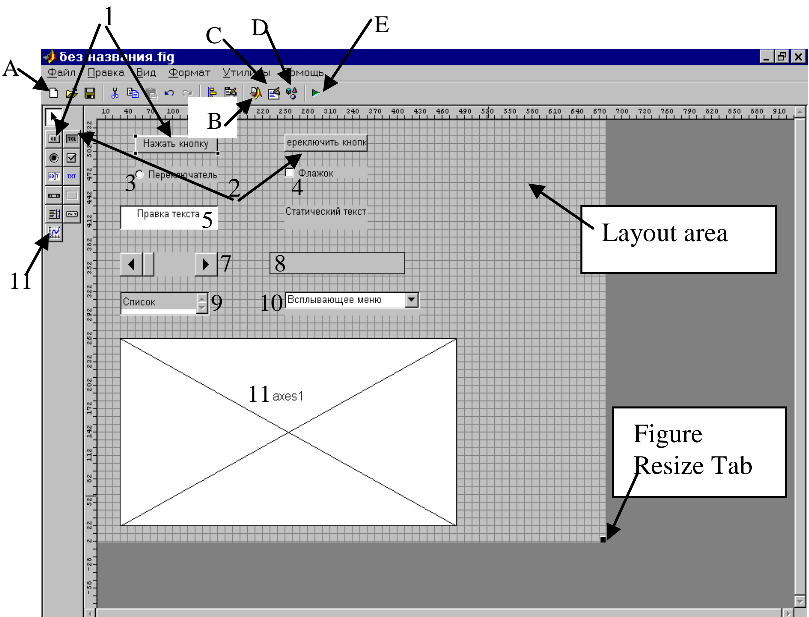


Fig. 4.4. GUI Editor with UI controls in the *Layout Area*:
A – *Property Editor* icon, B – *Run* icon; 1 – Push Button and its icon; 2 – Toggle Button; 3 – radio Button; 4 – Check box; 5 – Editable Text Box; 6 – Static Text; 7 – Slider; 8 – Frame; 9 – List box; 10 – Pop-Up Menu; 11 – Axes.

global ColorText
contents = get(hObject, 'String');
ColorText=contents{get(hObject, 'Value')};

The second command returns to variable all the contents of the *String*, but the third one gets the value selected from the pop-up menu. Again, the latter should be declared *global* here and in the code of the stage 9.

13. It is high time now to control speed of the rotation. In the function *SliderSpeed_Callback* that provides callback of the slider, write commands

global Speed
Speed=get(hObject,'Value')+.001;

that account for hints in comments there. The second command gets numeric value relevant to position of the slider. Note that default values for *Min=0* and *Max=1* have been not changed. This value is

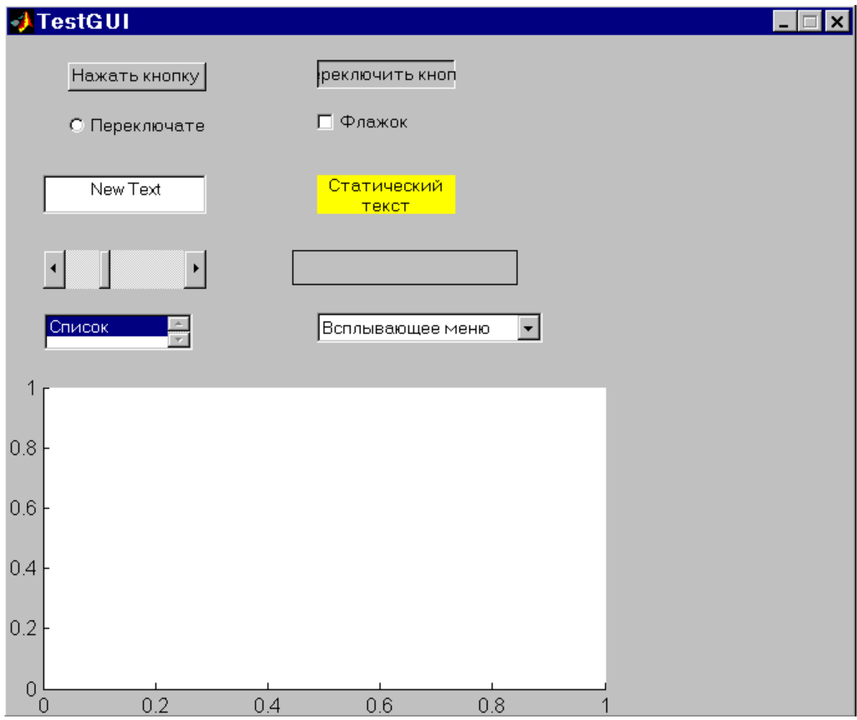


Fig. 4.5. Activated GUI shown in the Fig. 4.4. Each type of UI controls has been represented here. However, the UI controls "simulate" rather than execute a real work yet.

passed to the function *PushButtonStart_Callback* explained on the stage 9; it is suggested to uncomment the command before last.

At last, the graphical program *GUIhelicopter* has been completed. Execute the command *GUIhelicopter* from the Command Line and type in data it asks for. The program works fine. Figure 4.6 illustrates how the program works. Enjoy!

Conclusion

Before shaking hands, few words should be told to conclude. You learned MATLAB from the two viewpoints, as a mathematical environment in Modules 1 and 3, and as a programming tool in Modules 2 and 4. These both parts equally constitute this wonderful software and

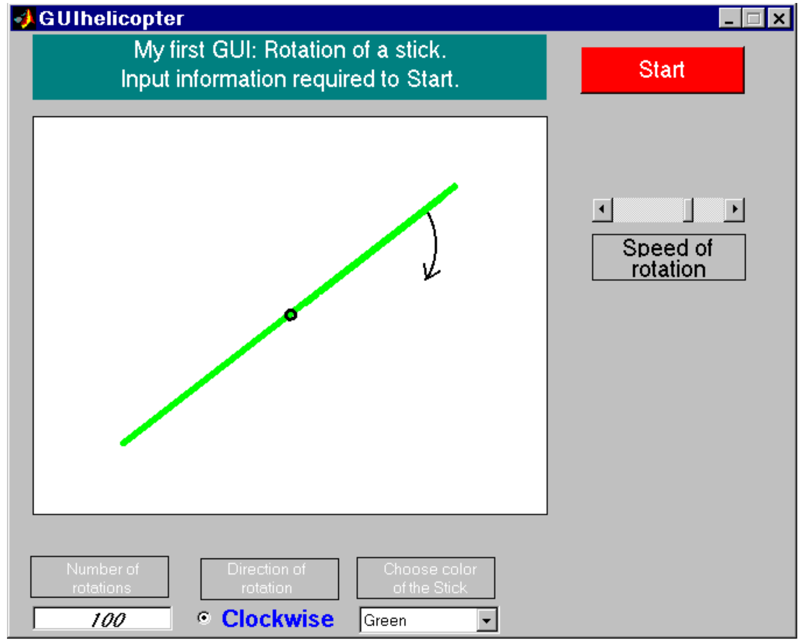


Figure 4.6. Graphical program *GUIhelicopter* that revolves stick of a chosen color *N* times in clockwise or opposite direction at a given speed.

make it indispensable in your study and future profession of information technology, IT. Because of volume restriction, we touched only few aspects of this very advanced and powerful program. It is our hope that you go on in mastering MATLAB and programming with it.

Problems for the Module 4

4.1. Give an example of soft you like and describe what of GUI it has.

4.2. Learn yourselves what perspectives in customizing your MATLAB GUIs may be provided by commands *uigetfile*, *uigetdir*, *helpdlg*, *uisetcolor*, *uisetfont*, *waitbar*, *questdlg*, *msgbox*, *warndlg*, *errordlg*.

4.3. Design a GUI program that creates propeller (1.9), fills it in with a specified color and rotates in one or opposite direction a specified number of times. Hints: use *plot* rather than *ezplot* and command *fill*; a subprogram for converting coordinates (x,y) to ones (x',y') rotated at an angle α might be useful.

4.4. Design a GUI program that creates a polygon with *N* vertices, fills it in with a color and rotates at various speeds (GUI shell to problem 2.10).
4.5. Develop a GUI program that asks for a function of one variable, finds its derivative and compares graphs of both (compare with problem 2.17).
4.6. Develop a GUI shell for program *limit1* (listing 3.1) that asks for a infinite numeric sequence like in problems 3.1 – 3.3, investigates existence of its limit and finds the latter if any.
4.7. Develop GUI program to demonstrate how Taylor series (Fourier series) may approximate their origin function (note the so called *Gibbs phenomenon* at discontinuity points in Fourier case), problems 3.4 – 3.7.

References

1. **Ануфриев И.** Самоучитель MatLab 5.3/6.x. – СПб.: БХВ-Петербург, 2002. – 736 с.
2. **Гаєв Є.О., Нестеренко Б.М.** Універсальний математичний пакет MatLab і типові задачі обчислювальної математики. Навчальний посібник. К., 2004. – 175 с.
3. **Гаєв Є.О., Нестеренко Б.М.** Типові задачі обчислювальної математики з застосуванням пакету MatLab. Методичні вказівки до виконання лабораторних робіт. К., 2004. – 38 с.
4. **Демидович В.П., Марон И.А.** Основы вычислительной математики. – М.: Наука, 1966. – 664 с.
5. **Денисюк В.П., Репета В.К., Гаєва К.А., Клешня Н.О.** Вища математика. Навч. посіб., ч. 3. К.: НАУб 2006. – 444с.
6. **Кетков Ю., Кетков А.** MatLab 6.x: программирование численных методов. Спб: БХВ, 2004. 672 с.
7. **Пайтген Х.-О., Рихтер П.Х.** Красота фракталов. – Москва: Мир, 1993. 176 с.
8. **Павловский В.И.** Структуры данных. Представление и использование. – Чернигов: ЧГТУ, 2003. – 233 с.
9. **Aho A.V., Hopkroft J.E., Ulman J.D.** Data structures and algorithms. (Russian translation: **Ахо А.В., Хопкрофт Д.Э., Ульман Д.Д.** Структуры данных и алгоритмы. – Москва-Киев: Изд. дом. "Вильямс", 2003. – 384 с.)

10. **Austin M., Chancogne D.** Introduction to Engineering Programming: in C, MATLAB, and Java. John Wiley&Sons, Inc., 1999.
11. **Azemi A., Yaz E. E.** Using graphical user interface capabilities of MATLAB in advanced engineering courses, *The 38th IEEE Conference on Decision and Control* (*CDC*), *IEEE*, pp 359-363, Phoenix, December 7-10, 1999.
12. **Cooper J.** A MATLAB companion for multivariable calculus. – San Diego: Harcourt, 2001. – 294 pp.
13. **Depcik Ch., Assanis D.N.** Graphical User Interfaces in an Engineering Educational Environment. *CAEE*2005, v13, №1, pp.48-59.
14. **Herniter M. E.** Programming in MATLAB**.** Thomson Engineering, 2001
15. **Rojan Yu.** Learn Programming and Mathematics with MATLAB.
16. **Tuck M.** The Real History of the GUI. http://www.sitepoint.com/article/ real-history-gui
17. **Wikipedia**, Graphical User Interface. http://en.wikipedia.org/wiki/Graphical_user_ interface
18. **Wirt N.** (Вирт Н. Алгоритмы и структуры данных. СпБ..: Изд-во "Невский диалект", 2001. – 352 с.)
19. **Gayev Ye.A., Nesterenko B.N.** MATLAB for Math and Programming: Textbook. – Zaporozhye: Polygraph, 2006 – 102 p.
20. **Азарсков В.М., Гаєв Є.О.** Сучасне програмування. Модулі 1,2: "Програмування та математика із другом MATLABом". К.: НАУ, 2014. – 256 с.
21. **Гаєв Є.О., Рожок О., Овчарчин Н.** Звук та музика в курсі програмування. -- Інженерія програмного забезпечення, 2014, (у друку).
22. **Гаев Е.А., Мартич М., Тарак Г.** Программы моделирования случайных явлений для изучения программирования и математики. – Інформаційні технології в освіті, 2015, (в печати)

Attachment A1

Listing of "*MyDiff.m*"

```
function [P,X1]=MyDiff(X,Y)
% Return Derivative of the table function Y=Y(X)
% provided length(X)=length(Y).
% Copyright Ye.Gayev
% (example of switch …case… end to compare with listing 2.2).
Nx=length(X); Ny=length(Y);
switch Nx==Ny
case 0
    disp('Error: Lengths of arguments X and Y must be equal!');
case 1
   disp('Differentiation has been completed!');
   disp('Call "plot(X1,P)" to plot the derivative.');
   X1=X(1 : end-1); % Coordinates of the derivative P(x)
   X2=X(2 : end); dX=X2-X1;
   Y1=Y(1 : end-1); Y2=Y(2 : end); dY=Y2-Y1;
   P=dY ./ dX;
otherwise
    % Nothing else except two above cases considered may happen
end
```

Attachment A2

Listing of "*GUIhelicopter.m*"

```
function varargout = GUIhelicopter(varargin)
% Portions I to III of this program  generated by MATLAB automatically
% has been skipped as they have been not the objects to edit

% Portion IV of the program:
% strings of code added by authors are written bold

%--- Executes on button press in PushButtonStart.
function PushButtonStart_Callback(hObject, eventdata, handles)
% hObject    handle to PushButtonStart (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
%N=5; Direction=1; ColorText='red';
global N Direction  ColorText  Speed
%ColorText
Dt=Direction*pi/20; % Determines speed of Rotations
for t=0:Dt:Direction*2*N*pi
    x1=cos(t); y1=sin(t);
    x2=-x1; y2=-y1;
```

```
% Pl=plot([x1, x2], [y1, y2], ColorText); set(Pl, 'linewidth',4)
% Colour of the Helicopter = Red now
    Pl=plot([x1, x2], [y1, y2]); set(Pl, 'linewidth',4); set(Pl, 'Color',ColorText)
   axis([-1.1 1.1 -1.1 1.1]) %Constant Scales!
   set(gca,'XTick', [  ]); set(gca,'yTick', [  ])
   hold on; plot([0] , [0],'o'); hold off; % Symbol and Colour of Centre Point
   pause(.1/Speed) % Speed of Rotations
end


% --- Executes during object creation, after setting all properties.
function EditN_CreateFcn(hObject, eventdata, handles)
% hObject    handle to EditN (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc
   set(hObject,'BackgroundColor','white');
else
   set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
end

function EditN_Callback(hObject, eventdata, handles)
% hObject    handle to EditN (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of EditN as text
%        str2double(get(hObject,'String')) returns contents of EditN as a double
global N
Ntext=get(hObject,'String');
N=str2double(get(hObject,'String'));


% --- Executes on button press in RadioButtonDirection.
function RadioButtonDirection_Callback(hObject, eventdata, handles)
% hObject    handle to RadioButtonDirection (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of RadioButtonDirection
global Direction
Direction=(-1)^get(hObject,'Value');
```

```
% --- Executes on selection change in PopUpColor.
function PopUpColor_Callback(hObject, eventdata, handles)
% hObject    handle to PopUpColor (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: contents = get(hObject,'String') returns PopUpColor contents as cell array
%        contents{get(hObject,'Value')} returns selected item from PopUpColor
global ColorText
contents = get(hObject,'String');
ColorText=contents{get(hObject,'Value')};


% --- Executes during object creation, after setting all properties.
function SliderSpeed_CreateFcn(hObject, eventdata, handles)
% hObject    handle to SliderSpeed (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: slider controls usually have a light gray background, change
%       'usewhitebg' to 0 to use default.  See ISPC and COMPUTER.
usewhitebg = 1;
if usewhitebg
    set(hObject,'BackgroundColor',[.9 .9 .9]);
else
    set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
end

% --- Executes on slider movement.
function SliderSpeed_Callback(hObject, eventdata, handles)
% hObject    handle to SliderSpeed (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'Value') returns position of slider
%        get(hObject, 'Min') and get(hObject,'Max') to determine range of slider
global Speed
% Min=get(hObject,'Min'), Max=get(hObject,'Max'),
Speed=get(hObject,'Value')+.001;


% --- Executes during object creation, after setting all properties.
function PopUpColor_CreateFcn(hObject, eventdata, handles)
% hObject    handle to PopUpColor (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
```

```
% handles    empty - handles not created until after all CreateFcns called

% Hint: listbox controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc
    set(hObject,'BackgroundColor','white');
else
    set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
end
% End of the program GUIhelicopter
```

Attachment A3

Listing of "*MyClock.m*"

```
%Program, that makes arrow-hands of a clock,
% synchronised  with your Windows-clock.
% The Clock will run infinitely.
%To stop it, press <Ctrl+C>
%Copyright Kornilov V.I., FCS-110 of NAU, Kyiv, December 2005

for i=1 : inf
    warning off MATLAB:divideByZero;
    time=fix(clock);
    ezplot('x^2+y^2=25'), hold on, axis equal; axis([-6  6  -6  6]), title(datestr(now));
    marksFi=[0 : pi/6 : 2*pi];
    marksRho=[0 : 1 : 12].*0+5;
    marksRho2=[0 : 1 : 60].*0+5;
    marksFi2=[0:pi/30:2*pi];
    plot(marksRho.*cos(marksFi), marksRho.*sin(marksFi), 'kh');
    plot(marksRho2.*cos(marksFi2), marksRho2.*sin(marksFi2), 'm.')
    plot(0, 0, 'ko')

    FiH=[pi/2    pi/2    -pi/20+pi/2    pi/2    pi/20+pi/2    pi/2    pi/2]-((pi/6)*time(4))-
((pi/6)*(time(5)/60));
    RhoH=[0 3 3 4 3 3 0];
    xH=RhoH.*cos(FiH);
    yH=RhoH.*sin(FiH);
    fill(xH,yH,'r'),  %pause(.01);

    FiM=[pi/2    pi/2    -pi/32+pi/2    pi/2    pi/32+pi/2    pi/2    pi/2]-((pi/30)*time(5))-
((pi/30)*(time(6)/60));
    RhoM=[0 2 2 5 2 2 0];
    xM=RhoM.*cos(FiM);
    yM=RhoM.*sin(FiM);
    fill(xM,yM,'c'), %pause(.01);
```

```
    FiS=[pi/2 pi/2 -pi/64+pi/2 pi/2 pi/64+pi/2 pi/2 pi/2]-(pi/30)*time(6);
    RhoS=[0 1 1 5 1 1 0];
    xS=RhoS.*cos(FiS);
    yS=RhoS.*sin(FiS);
    fill(xS,yS,'b'), hold off, pause(1);
    WatchArr(:,i) = getframe;
end;
```

Summary of MATLAB commands

Command name	Explanation	Page
asin	$arcsin(x)$	23
atan	$arctg(x)$	22
axis	scaling and appearance of axes	13,14,35,42,81,89
\bf	example of LaTeX representation of a symbolic expression	54,55,59
cftool	separate Curve Fitting Tool	64
clear	Clearing variables from memory	54,66
clock	Getting current date and time	26,38,39
collect	Collecting coefficients of symbolic expression	19
comet	Plotting comet-like trajectory	14,26
cos	cos(x)	
<Ctrl+C>	COPY hotkey, or breaking execution if computer hangs	35,73,79,91
<Ctrl+V>	PASTE hotkey	79
date	Getting current date	26
det	$\mid A \mid$, determinant of square matrix	19,23
diag(x)	diagonal square matrix with the elements from x	14
diff	differentiation, either numeric vector of symbolic function	21

disp	displays content of argument	38,43-50,58-61,88
else *elseif*	key words of i*f* ... *end* statement condition	35,37-40,54,89-91
end	key word of array last element number, or end of a structural *for*-, *while*-, *if*- etc. blocks	
expand	removes brackets of products of symbolic expressions	18,19
eval	evaluates string as MATLAB expression and executes it	22,23
eye	producing Identity matrix	10
ezplot *ezsurf*	easy-to-use plotting commands in 2 and 3 dimensions	13,15,20-22,37,49,59
false	logical zero, *0*	34,39-41,44
figure	create new figure window	13,22,28,49,63,75
fill	creation of 2d polygon filled by a specified colour	15,51,84
for	key word of execution loop	34,35,43,45-47,46
format	setting format of outputting numeric data	
fplot	easy-to-use 2d plotting	13,26,38,42,51,59
function	key word for be adding a new command to MATLAB's vocabulary	29,31,37,41,43,45-47
fzero	tries to find a zero of a function if an initial guess is provided	23

gcf	getting handle to current figure	73
get	getting properties of an object	73,75,81-83,89,90
global	declaration for exchange by variables between programs	32,33,81-83,88-90
grid	drawing grid lines on a plot	26,28
guide	program for designing GUI	76
help	enquiring HELP from command line	10,11,30,34,37
i	$\sqrt{-1}$	8
if	key word of conditional program block	34-40,43-45,54,89-91
inf	MATLAB's notation of ∞	
input	prompting for user input	48,49,54,58
int	command for integrating symbolic function	
inv	calculate inverse matrix	24
j	$\sqrt{-1}$	8
legend	puts legends of current plotted curves	24,67
length	determination of vector length	12,43,45,46,62,88
max	determines largest component of vector argument, or property of a Figure object	46,80,83,90
menu	generates Figure with a menu of choices	71-75

min	determines smallest component of vector argument, or property of a Figure object	46,80,83,90
NaN	Not-a-Number, results of mathematically undefined operations	
ones	matrix with only 1s	10
pause	pauses program execution	35,48,49,58,89
pi	$\pi = 3.14159...$	
plot	plots vector versus vector	14,28,35,42,54,62,63,65,67,81,88,89
plot3	a three-dimensional analogue of *plot*	
poly	converts vector of roots to vector of polynomial coefficients	
poly2sym	converts polynomial from numeric to symbolic representation	21
polyval	evaluates polynomial at given *x*	18
pretty	prints symbolic expression on screen in LaTeX form	19,21,23
rand *randn*	generates random numbers distributed uniformly or normally	61,62,64,66
roots	finds roots of numerically represented polynomial	17,18
round	rounding data towards nearest integer	66
set	setting properties of Figures	35,73-76,81,89-91

size	returns both dimensions of matrices	12,47
sqrt	$\sqrt{\quad}$	
solve	solves algebraic equation sets symbolically or numerically	19,23
subs	substitutes symbols in symbolic expressions	20,21,23
sum	sums elements of vector	12
switch	key word of *switch…case…otherwise…end* statement	35,46,88
sym, syms	declarations of symbolic variables	18,21,23,48
sym2poly	converts polynomial from symbolic to numeric representation	20
taylor	finds Taylor series expansion of given symbolic function	23
text	annotating a text on Figures	54
tic *toc*	starting and reading stopwatch timer	65
title	printing title text on graphics	34,39-41
true	logical one, *1*	
uicontrol	creates user interface controls	74,75
while	key word of ***while…end*** statement looping indefinite number of times	35,43,44,54

xlabel *ylabel*	labelling *Ox* and *Oy* axes on Figures	28,54,67
zeros	matrix with only zeros, *0*	10

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Author’s address
Адрес автора Ye_Gayev@voliacable.com